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Preparation 10

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Shape and Space

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Module 5

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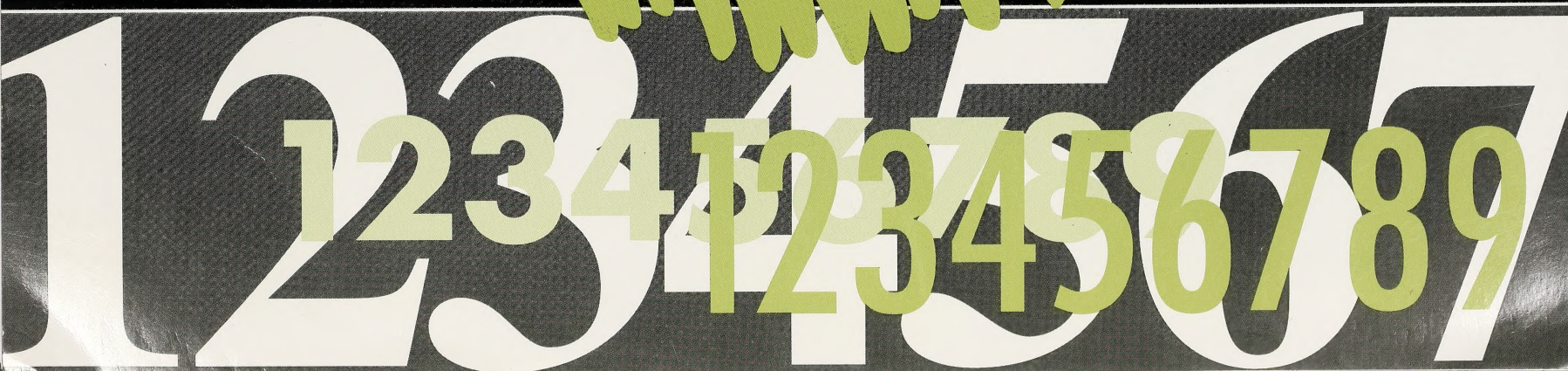

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MATHEMATICS
Preparation 10

Module 5
Shape and Space

Mathematics Preparation 10
Module 5: Shape and Space
Student Module Booklet
Learning Technologies Branch
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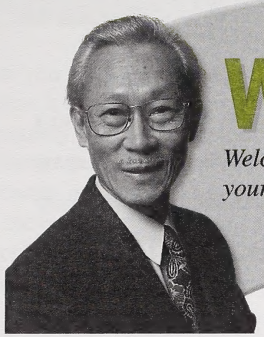

Welcome

Welcome to Module 5. We hope you enjoy your study of "Shape and Space."

Course Overview

Mathematics Preparation 10 consists of five modules and a final test. Each module is worth 1 credit. Discuss with your teacher how many modules you need to complete to prepare yourself for Pure Mathematics 10 or Applied Mathematics 10.

Mathematics Preparation 10

Module 1
Exploration of Numbers

Module 2
Number Connections

Module 3
Patterns and Algebra

Module 4
Polynomials

Module 5
Shape and Space

The document you are now reading is called a Student Module Booklet. It has accompanying Assignment Booklets.

You will find many visual cues or icons throughout this Student Module Booklet. Read the following statements to discover what each icon prompts you to do.

Use the Internet to explore a topic.

Use a scientific calculator.

Use the suggested answers in the Appendix to correct activities.

Answer the questions in the Assignment Booklet.

Throughout the course, you will be given instruction and practice on each of the following mathematical processes:

- connecting mathematical ideas
- developing and using estimation strategies
- developing and using mental math strategies
- using technology appropriately
- developing and using problem-solving strategies
- reasoning and justifying your answers
- using visualization to assist in processing information
- solving problems
- communicating mathematically

For example, to help you develop your problem-solving skills, several problem-solving strategies are explained in the Appendix of each Student Module Booklet, and you will be given several non-routine problems to solve in each module. Watch for the heading "Now Try This."

To help you develop your communication skills in mathematics, you will be asked to do several journal writings in each module. Watch for the heading "Looking Back."

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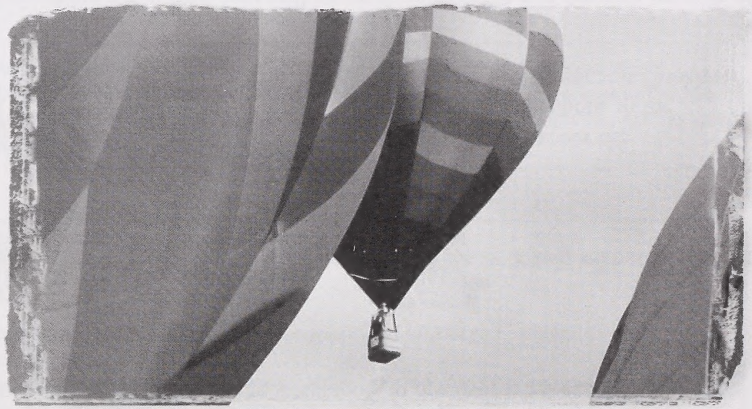


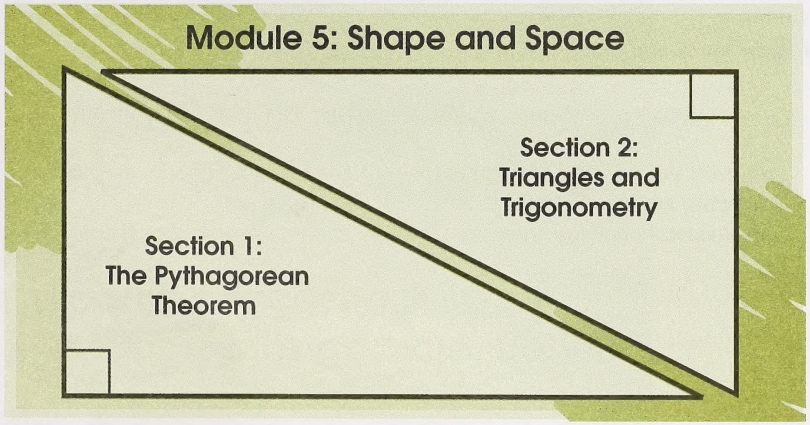
NOVA DEVELOPMENT CORPORATION

Module Overview

Did you perform some type of measurement today? Measurement in various forms is a vital part of everyday life, as is knowledge of certain aspects of geometry. Everything—from the right amount of toothpaste, to the shortest distance to a friend's house, to the ability to draw in perspective—involves accurate measurement and a grasp of certain geometric concepts. You do many of these tasks without consciously thinking of measuring because they have become automatic through repetition. Other tasks may require thoughtful consideration and careful calculations.

In Section 1, you will work with squares and square roots and use the Pythagorean Theorem.

In Section 2, you will study similar triangles and use this knowledge to enlarge and reduce figures. You will expand your knowledge of right triangles by examining the ratios of the lengths of the sides of a right triangle. These trigonometric ratios will help you solve more problems involving right triangles.

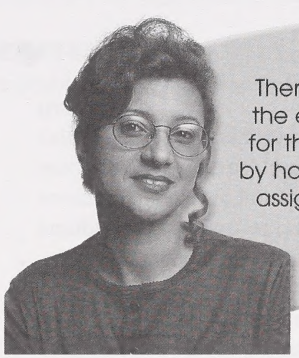


Assessment

Your mark for this module will be determined by how well you complete the assignments at the end of each section. In this module, you must complete three assignments. The mark distribution is as follows:

Assignment Booklet 5A	
Section 1 Assignment	**20 marks**
Assignment Booklet 5B	
Section 2 Assignment	**40 marks**
Final Module Assignment	**40 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently; but if you are having difficulties, you may review the appropriate section in this Student Module Booklet.



There is a supervised final test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Stragegies for Completing the Module Successfully

To achieve success in this module, be sure to work slowly and systematically through the Student Module Booklet. Remember, your work in the Student Module Booklet will prepare you for your module assignments and final test.

Following are some strategies for completing the module successfully:

- Try to set realistic goals for each day. Once you've set these goals, stick to them.
- Read all the instructions in each Student Module Booklet carefully; answer all the questions in your mathematics binder; and check your answers by comparing your responses with the suggested answers in the Appendix.
- Keep a section of your mathematics binder for your journal entries. Get in the habit of describing new concepts, procedures, and strategies in your own words. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, and charts) to help you connect mathematical ideas.
- Ask someone who is taking Mathematics Preparation 10 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.
- If you need assistance from your teacher or your study partner, you may find it helpful to write your questions in the journal section of your mathematics binder before speaking to your teacher or study partner. This will help you pinpoint your problem.
- Take care when completing your module assignments. Be sure you have completed each part of the Assignment Booklets, and proofread the assignments before you submit them. Remember to review your module assignments after they are corrected; then file them in your binder so you can review your assignments and mathematics notes before you write the final test.

Good luck!

SECTION 1

The Pythagorean Theorem

The annual flooding of the Nile River forced the ancient Egyptians to survey property boundaries every year. One of the survey tools they used to determine property boundaries was a knotted rope. This rope would be stretched out and formed into a triangle with sides in the ratio $3:4:5$. The Egyptians knew that a triangle with sides safisfying this ratio formed a right angle. This is just one example of the Pythagorean Theorem, $a^2 + b^2 = c^2$, which holds for all right triangles. The relation $3^2 + 4^2 = 5^2$ was an Egyptian rule of thumb long before 540 B.C., when Pythagoras first proved the theorem named after him.

The Pythagorean Theorem is used frequently in mathematics. In fact, many of the fundamental formulas for trigonometry are based upon the Pythagorean Theorem.

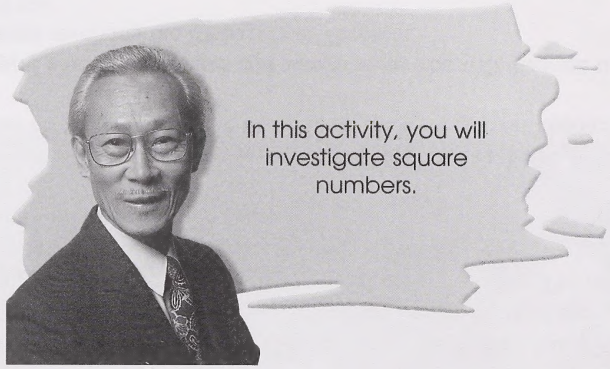
In this section, you will work with squares and square roots. You will then use these skills to solve problems involving the Pythagorean Theorem.

Activity 1: Squares and Square Roots

Some ancient Greek mathematicians spent considerable time arranging dots into geometric shapes and then counting the dots.

They discovered that some numbers, such as 1, 3, 6, and 10, are **triangular numbers**.

They also discovered that some other numbers, such as 1, 4, 9, and 16, are **square numbers**.



The following is a sequence of the first ten square numbers:

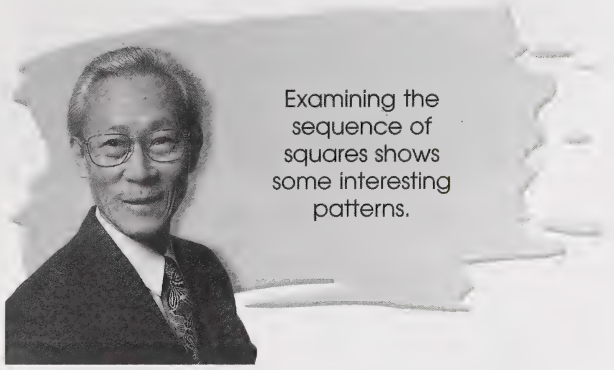
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, …

The sequence of square numbers was formed by multiplying the consecutive counting numbers by themselves.

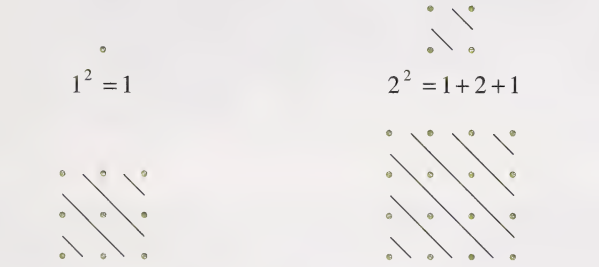
1,	4,	9,	16,	25,	36,	49,	64,	81,	100, …
↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
1×1	2×2	3×3	4×4	5×5	6×6	7×7	8×8	9×9	10×10

The sequence of square numbers can be written using exponential notation.

$1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, \ldots$



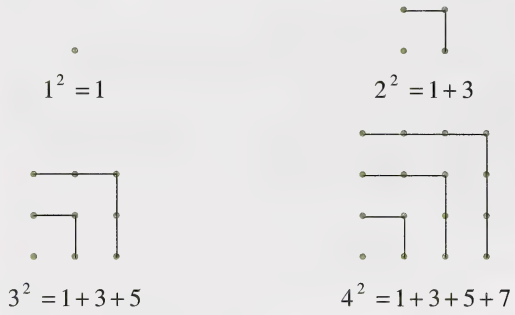
1. Look at the sequence of square numbers when it is arranged as follows:



a. Draw a similar diagram for 5^2, and write a number sentence to describe the square.

b. Use the pattern you have discovered to write a sentence to describe 10^2.

2. Look at the sequence of square numbers when it is arranged as follows:

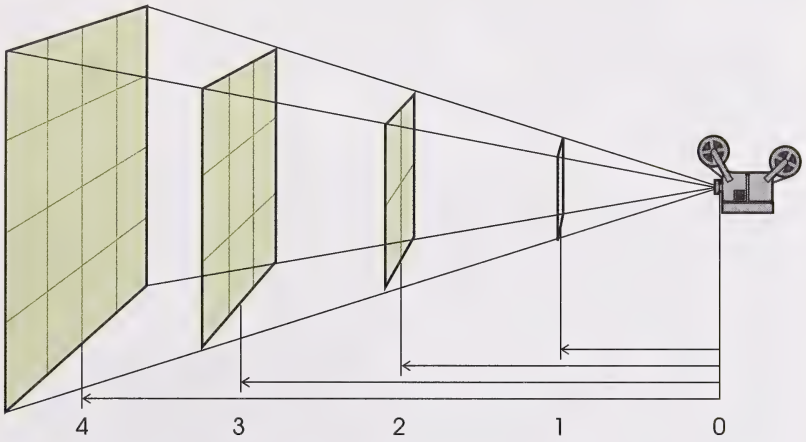


a. Draw a similar diagram for 5^2, and write a number sentence to describe the square.

b. Draw a similar diagram for 6^2, and write a number sentence to describe the square.

c. Write a number sentence to describe 10^2. **Note:** Do not draw a diagram.

3. Walter was watching home movies with his family. He noticed that the area of the picture on the screen was related to the distance the projector was from the screen. (This relationship is illustrated in the following diagram.)



a. Use the diagram to complete the following table.

Distance from Screen (units)	Area of Picture (square units)
1	
2	
3	
4	

b. Write a formula to describe how the area of the picture on the screen is related to the distance of the projector from the screen. Let a be the area (in square units) of the picture, and let d be the distance (in units) from the screen.

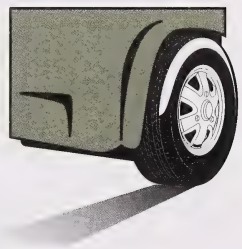
c. Use the formula to calculate the area of the picture on the screen if the projector is 5 units from the screen.

4. The approximate distance an object will fall when it is dropped is given by the following formula, where d is the distance (in metres) and t is the time (in seconds).

$$d = 5t^2$$

An object is dropped from the top of the CN Tower in Toronto. How far will the object fall in the first 4 s?

5. The approximate braking distance of a car on dry pavement is given by the following formula, where d is the braking distance (in metres) and s is the speed (in kilometres per hour).

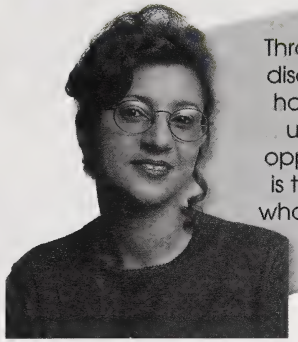


$$d = \frac{s^2}{210}$$

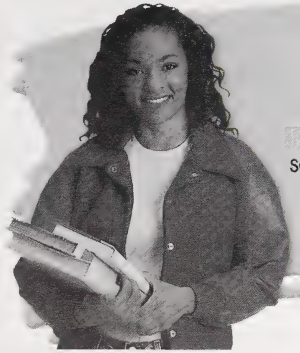
a. What is the braking distance of a vehicle travelling 50 km/h?

b. What is the braking distance of a vehicle travelling 100 km/h?

Check your answers by turning to the Appendix, page 91.



Throughout this course, you have discovered that every operation has an opposite operation that undoes it. If subtraction is the opposite of addition and division is the opposite of multiplication, what is the opposite of squaring a number?



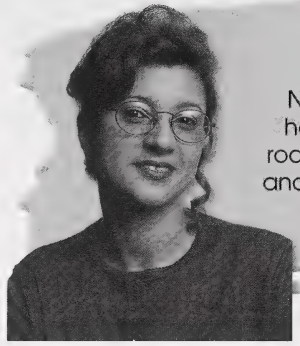
To undo the operation of squaring, find the square root.

A square root is one of two equal factors that yields a product equal to a given number.

For example,

$$2^2 = 2 \times 2 \quad \text{and} \quad (-2)^2 = (-2) \times (-2)$$
$$= 4 \qquad\qquad = 4$$

Therefore, the square roots of 4 are 2 and -2.



Numbers always have two square roots: a positive root and a negative root.

The symbol $\sqrt{4}$ represents the **positive** square root of 4. The positive square root is also called the **principal** square root.

$$\sqrt{4} = 2$$

The symbol $-\sqrt{4}$ represents the **negative** square root of 4.

$$-\sqrt{4} = -2$$

Example

What number squared is equal to 64?

Solution

Let n be the number.

$$\therefore\ n^2 = 64$$
$$n = \sqrt{64} \text{ or } -\sqrt{64}$$
$$= 8 \text{ or } -8$$

The square root of 64 is 8 or -8, since $8 \times 8 = 64$ and $(-8) \times (-8) = 64$.

Example

If the area of a square is $36\ \text{cm}^2$, what is the measure of one side?

Solution

Use the formula $A = s^2$.

$$A = s^2$$
$$36 = s^2$$
$$s = \sqrt{36}$$
$$= 6$$

Because length is always positive, use only the positive square root.

The measure of one side is 6 cm.

6. Evaluate the following.

a. $\sqrt{100}$ **b.** $\sqrt{256}$ **c.** $-\sqrt{81}$

d. $-\sqrt{400}$ **e.** $\pm\sqrt{121}$ **f.** $\pm\sqrt{144}$

Check your answers by turning to the Appendix, page 92.

A perfect square is a number that has a natural number as its principal square root. The natural numbers, sometimes called the counting numbers, are 1, 2, 3, 4, 5, 6, … .

A calculator is a valuable tool when finding the decimal approximation of the square root of a number that is not a perfect square. Whenever you use a calculator to find the square root of a number, estimate first. One way to estimate is to use the perfect squares you know.

Example

Estimate $\sqrt{57}$. Then use a calculator to find the decimal approximation to the nearest tenth.

Solution

Estimate

Pick perfect squares below and above 57.

$$\sqrt{49} < \sqrt{57} < \sqrt{64}$$
$$7 < \sqrt{57} < 8$$

Because 57 is approximately halfway between 49 and 64, $\sqrt{57} \doteq 7.5$.

Decimal Approximation

$\therefore \sqrt{57} \doteq 7.5$

Example

Estimate each of the following. Then use a calculator to find the decimal approximation to 2 decimal places.

a. $\sqrt{3543}$ **b.** $-\sqrt{0.0845}$

Solution

a. **Estimate**

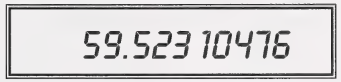
Round 3543 to the nearest 100.

$$3543 \doteq 3500$$

$$\therefore \sqrt{2500} < \sqrt{3500} < \sqrt{3600}$$
$$50 < \sqrt{3500} < 60$$

Because 3500 is close to 3600, $\sqrt{3543} \doteq 60$.

Decimal Approximation



$\therefore \sqrt{3543} \doteq 59.52$

b. Estimate

Round 0.0845 to the nearest hundredth.

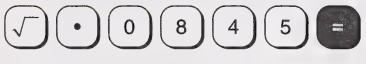
$0.0845 \doteq 0.08$

$\therefore \sqrt{0.04} < \sqrt{0.08} < \sqrt{0.09}$

$0.2 < \sqrt{0.08} < 0.3$

Because 0.08 is close to 0.09, $-\sqrt{0.0845} \doteq -0.03$.

Decimal Approximation



$\therefore -\sqrt{0.0845} \doteq -0.29$

Use a scientific calculator to answer question 7.

7. Estimate each of the following; then find the decimal approximation to 2 decimal places.

a. $\sqrt{20}$ **b.** $\sqrt{7}$ **c.** $\sqrt{1215}$

d. $-\sqrt{32}$ **e.** $-\sqrt{15}$ **f.** $\sqrt{873}$

g. $\pm\sqrt{0.006}$ **h.** $\pm\sqrt{23\,815}$ **i.** $\pm\sqrt{0.0024}$

8. Use the information from the answers to question 7 to answer the following questions.

a. What did you notice about the number of digits in each calculator display?

b. If the calculator could display more digits, would there be more digits in each display?

Check your answers by turning to the Appendix, page 92.

Looking Back

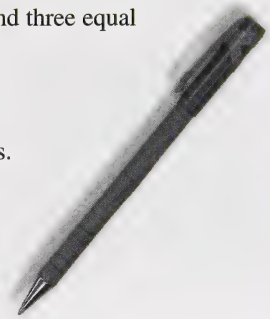
In this activity, you worked with squares and square roots. You estimated square roots of numbers using your knowledge of perfect squares.

You can estimate cube roots $\left(\sqrt[3]{\ }\right)$ by the same method you used for estimating square roots. To determine a square root of a number, you needed to find two equal numbers whose product was the number. For example,

$\sqrt{25} = 5$ since $5 \times 5 = 25$

$\sqrt{625} = 25$ since $25 \times 25 = 625$

To determine a cube root of a number, you need to find three equal numbers whose product is the number. For example,
$\sqrt[3]{8} = 2$ since $2 \times 2 \times 2 = 8$.



9. In your journal, write the next four perfect cubes.

1^3, 2^3, 3^3, 1, 8, 27, …

10. Using your list of numbers from question 9, estimate the following cube roots. An example has been done for you.

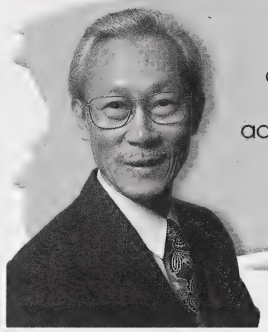
$\sqrt[3]{9}$ Because 9 lies between 8 and 27, $\sqrt[3]{9}$ lies between 2 and 3 and is closer to 2.

a. $\sqrt[3]{100}$ Because _____ lies between _____ and _____, $\sqrt[3]{100}$ lies between _____ and _____ and is closer to _____.

b. $\sqrt[3]{36}$ Because _____ lies between _____ and _____, $\sqrt[3]{36}$ lies between _____ and _____ and is closer to _____.

Check your answers by turning to the Appendix, page 95.

Activity 2: Equations Containing Squares



In Module 3 of this course, you solved equations. In this activity, you will look at solving equations involving squares.

Remember: As with any equation, you must maintain the equality by performing the same operation on each side of the equation.

Example

Solve $a^2 - 3 = 13$.

Solution

Add 3 to each side of the equation.

$$a^2 - 3 + 3 = 13 + 3$$
$$a^2 = 16$$

Take square roots of each side.

$a = \sqrt{16}$ or $-\sqrt{16}$

$= 4$ or -4

Recall that a positive number has two square roots: a positive root and a negative root. The square roots of 16, in this case, are 4 or -4.

Check

LS	RS
$a^2 - 3$	13
$= 4^2 - 3$	
$= 16 - 3$	
$= 13$	
LS =	RS

LS	RS
$a^2 - 3$	13
$= (-4)^2 - 3$	
$= 16 - 3$	
$= 13$	
LS =	RS

Therefore, $a = 4$ and $a = -4$ are both solutions to the equation.

1. Solve the following equations.

a. $x^2 + 1 = 10$

b. $r^2 - 1 = 24$

c. $3t^2 = 108$

d. $\frac{n^2}{2} = 32$

e. $25x^2 + 1 = 65$

f. $4b^2 - 3 = 33$

2. Write an equation that describes the situation; then solve.

a. Eight less than the square of a number is seventeen. What is the number?

b. Ten times the square of a number is forty. What is the number?

3. The mass of a freshly caught fish can be estimated using the following formula, where m is the mass (in kilograms), ℓ is the length (in centimetres), and g is the girth (in centimetres) of the fish.

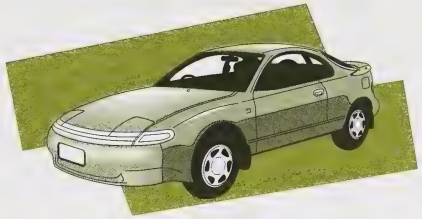
$$m = \frac{\ell \times g^2}{24\,600}$$

Girth is the distance around the thickest part of the fish.

What is the mass of a salmon that is 82 cm long with a girth of 50 cm? Round your answer to the nearest tenth.

4. The approximate braking distance for a car on dry pavement is given by the following formula, where d is the braking distance (in metres) and v is the rate of speed (in km/h).

$$d = \frac{v^2}{210}$$



a. Find the approximate braking distance at 80 km/h. Round your answer to 1 decimal place.

b. The police officer who arrived at the scene of a car accident measured the skid marks and found that the braking distance was 62 m. Calculate the speed of the vehicle. Round your answer to the nearest whole number.

5. A circle has an area of 153.7 cm^2. What is the radius of the circle to the nearest centimetre? Estimate your answer first. (**Hint :** $A = \pi r^2$)

6. **a.** Why did you only use the positive value of the radius in question 5?

b. What name is given to the positive value of a square root?

c. Does a calculator give the negative value for the square root of a positive number?

d. Try to take the square root of -49. What happens? Why?

Check your answers by turning to the Appendix, page 95.

In general, decimal numbers that do not repeat or terminate are known as irrational numbers. All square roots of non-perfect squares are irrational numbers. Some examples of irrational numbers are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, and $\sqrt{8}$. You will study more about irrational numbers in future mathematics courses.

7. Catherine has a square cross-stitch picture with sides measuring 24 cm. She wants to mount it on a piece of drymount board that has twice the area of the cross-stitch picture. Calculate the length of each side of the required piece of drymount board to the nearest centimetre. Estimate the length first.

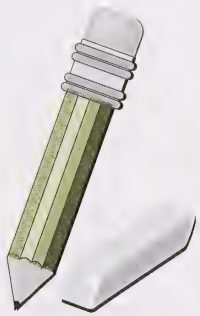
8. Gerald and Mabel own a mixed farm. They want to fence a square, 4-ha pasture for their sheep. How many metres of fencing are required for the pasture? ($1 \text{ ha} = 10\,000 \text{ m}^2$)

9. Suppose you know the area of a square. Explain, in words, how you would determine the perimeter of the square.

Check your answers by turning to the Appendix, page 97.

Looking Back

In this activity, you solved equations that contained a variable squared and used the inverse operation of squaring, which is taking the square root.



10. In your journal, do the following problems so you have examples of solving equations that contain squares.

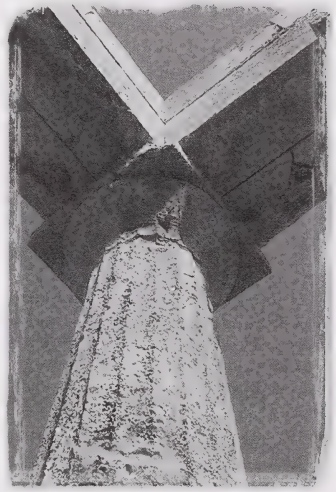
    a. Solve $3x^2 + 87 = 162$.

    b. The volume, V, of any cylinder is $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder. The volume of a particular cylinder with a height of 12 cm is approximately $340\ \text{cm}^3$. Calculate the radius to the nearest tenth.

Check your answers by turning to the Appendix, page 104.

Activity 3: Understanding and Applying the Pythagorean Theorem

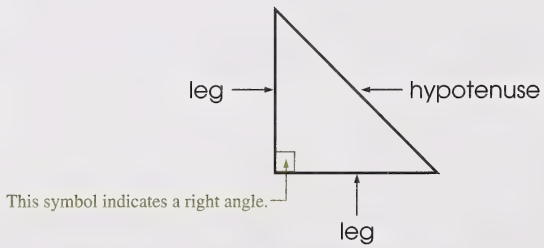
A mathematician and philosopher by the name of Pythagoras (approximately 575 B.C.–495 B.C.) proved a relationship among the lengths of the sides of a right triangle.



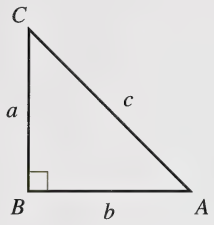
In this activity, you will examine this relationship and use it to model and solve problems involving right triangles.

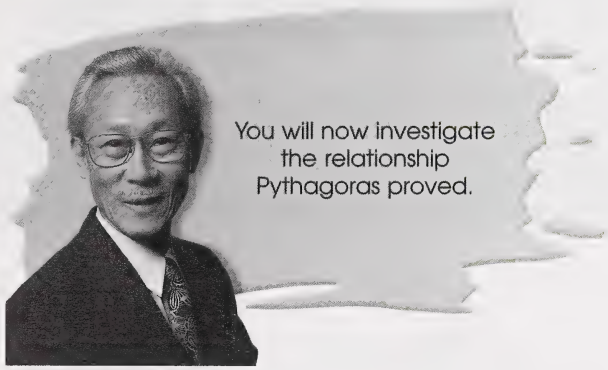
A right triangle is a triangle that has one angle measuring 90°.

The side opposite the right angle of a right triangle is called the **hypotenuse**. It is the longest side because it is across from the largest angle. The other two sides of the triangle are the **legs**.



Each side of a right triangle is often represented by a lower-case letter. The lower-case letters correspond to the upper-case letters representing the opposite angles. For example, side a lies opposite angle A.





At the back of the Appendix, find Pythagorean Puzzle 1. If you are photocopying this page, glue it to heavy paper or boxboard. You may find the pieces easier to handle.

Cut out Square A and Square B; then cut Square A along the dotted lines. Arrange Square B and the four pieces of Square A over Square C.

Notice how squares have been constructed on the legs and hypotenuse of each right triangle. In each triangle, Square A has a side of a; therefore, the area of Square A is a^2. Square B has a side of b; therefore, the area of Square B is b^2. Square C has a side of c; therefore, the area of Square C is c^2.

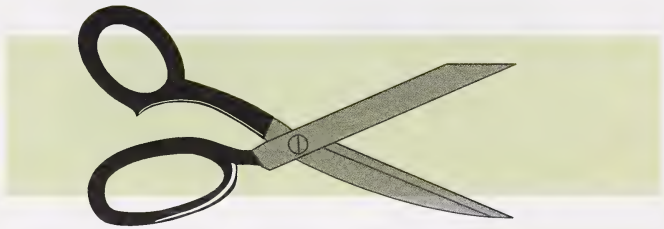
1. a. In Pythagorean Puzzle 1, do Square B and the four pieces of Square A cover Square C?

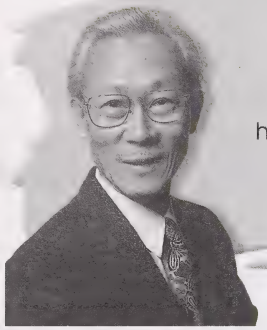
b. What conclusion can you make about the area of Square C? Write the relationship as an equation.

2. a. In Pythagorean Puzzle 2, do Square B and the four pieces of Square A cover Square C?

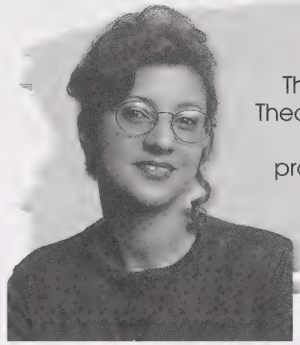
b. What conclusion can you make about the area of Square C? Write the relationship as an equation.

Check your answers by turning to the Appendix, page 98.



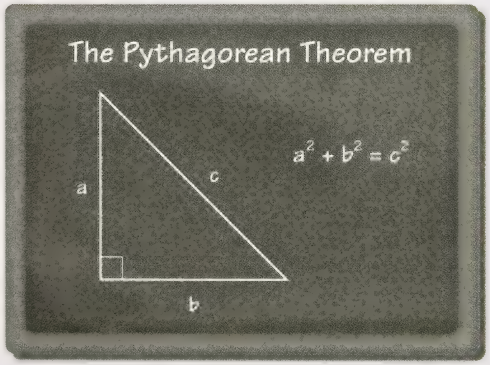


You have just discovered the reasoning that helped develop the Pythagorean Theorem.



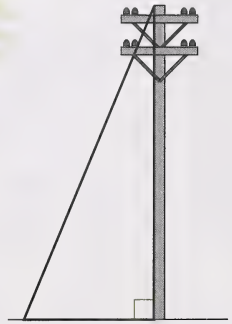
The Pythagorean Theorem can be used to solve many problems involving right triangles.

The Pythagorean Theorem is a theorem stating that the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.



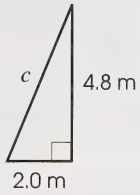
Example

A 4.8-m power pole is held up by a guy wire anchored 2.0 m from its base. How long is the guy wire?



Solution

Step 1: Draw and label a diagram to represent the problem.



Step 2: Write the formula for the Pythagorean Theorem, and substitute the given values. Then simplify the equation.

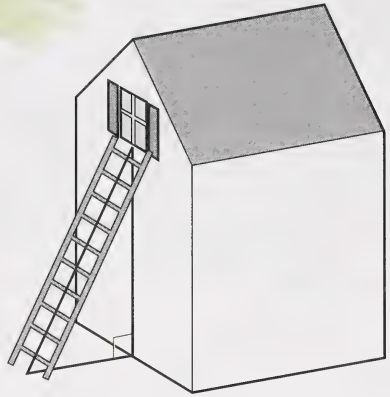
$$a^2 + b^2 = c^2$$
$$4.8^2 + 2.0^2 = c^2$$
$$23.04 + 4.0 = c^2$$
$$27.04 = c^2$$
$$c = \sqrt{27.04}$$
$$= 5.2$$

Because length is always positive, use only the positive square root.

The guy wire is 5.2 m long.

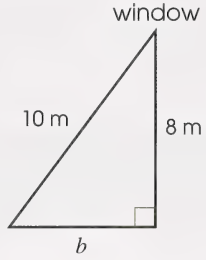
Example

A 10-m long ladder reaches a second-storey window. If the distance from the window sill to the ground is 8 m, how far is the foot of the ladder from the house?



Solution

Step 1: Draw and label a diagram to represent the problem.



Step 2: Write the formula for the Pythagorean Theorem, and substitute the given values. Then simplify the equation.

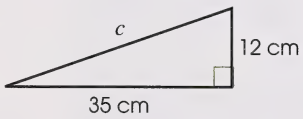
$$a^2 + b^2 = c^2$$
$$8^2 + b^2 = 10^2$$
$$64 + b^2 = 100$$
$$64 + b^2 - 64 = 100 - 64$$
$$b^2 = 36$$
$$b = \sqrt{36}$$
$$= 6$$

Because distance is always positive, use only the positive square root.

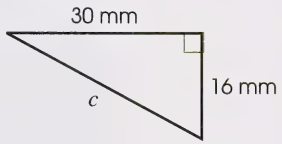
The foot of the ladder is 6 m from the house.

3. For each of the following triangles, calculate the length of the missing side. Round your answers to the nearest tenth where necessary.

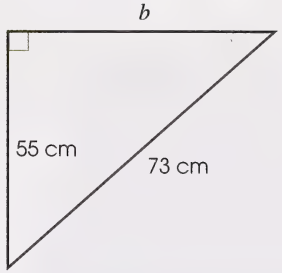
a.



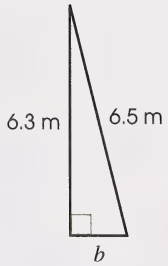
b.



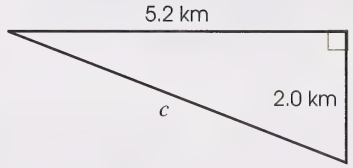
c.



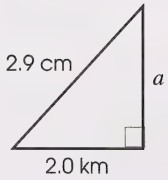
d.



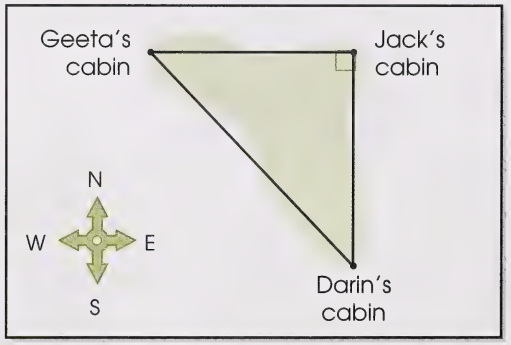
e.



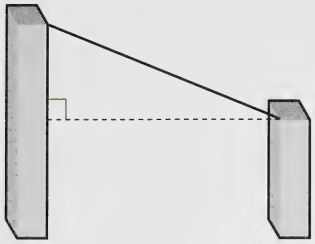
f.



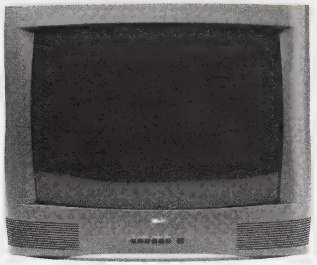
4. Darin's cabin is 2.4 km south of Jack's cabin. Geeta's cabin is 3.0 km north and west of Darin's cabin. Geeta's cabin is west of Jack's cabin. How far across the lake is Geeta's cabin from Jack's cabin?



5. Two walls are connected by a steel cable that is 13 m long. If one wall is 11 m high and the other is 6 m high, find the distance between the walls.



6. A television screen is a rectangle that measures 30 cm wide and 22 cm high. What is the diagonal measure of the screen? Round your answer to the nearest tenth.



7. A baseball diamond is a square with sides about 27.4 m. A player at second base must throw the ball to home plate. How far must the player throw the ball? Round your answer to the nearest tenth.

8. A box is 120 cm long and 25 cm wide. What is the length of the longest ski pole that could be packed to lie flat in the box? Round your answer to the nearest tenth.

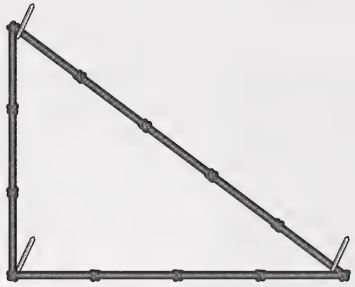
Check your answers by turning to the Appendix, page 99.

Did You Know?

Houses would look very strange if there were no way of making 90° angles. Fortunately, there are many tools to help builders make square corners.

The ancient Egyptians did not have modern tools. How do you think they made 90° angles?

To make a square corner, the Egyptians tied knots to divide a rope into 12 equal parts. The rope was then stretched and formed into a triangle with lengths of 3, 4, and 5 units. Stakes were used to hold the triangle securely.



Why did this method of making 90° angles work? The numbers 3, 4, and 5 make up a Pythagorean triple; and a triangle with sides of 3, 4, and 5 units is a right triangle.

Three numbers form a Pythagorean triple if the sum of the squares of the two smaller numbers equals the square of the largest number.

9. Is each of the following a Pythagroean triple? Answer **yes** or **no**.

   a. 5, 7, 10 **b.** 7, 24, 25 **c.** 9, 13, 14

   d. 5, 12, 13 **e.** 7, 20, 21 **f.** 1, 10, 11

10. One Pythagorean triple is 3, 4, and 5.

    a. Is 6, 8, and 10 a Pythagorean triple?

    b. Is 9, 12, and 15 a Pythagorean triple?

    c. Name another Pythagorean triple that is a multiple of 3, 4, and 5.

11. What seems to be a general rule for finding other Pythagorean triples once you know one?

Check your answers by turning to the Appendix, page 101.

Looking Back

In this activity, you studied the Pythagorean Theorem. You examined this theorem and used it to calculate lengths and solve a variety of practical problems.

12. In your journal, define the Pythagorean Theorem and write the formula. Use the words *leg* and *hypotenuse* in your definition.

Check your answers by turning to the Appendix, page 102.

Conclusion

In this section, you explored squares and square roots. As part of your exploration, you investigated the Pythagorean Theorem and its applications in a variety of practical situations.

The Pythagorean Theorem is used frequently by people in the construction industry, in architecture, and in design, just to name a few. Look around the room you are currently in. Right angles are everywhere, in particular where the walls meet the ceiling and floor. How accurate are these angles? When buildings are constructed, it is important that the angles and other measurements are accurate so other components will fit together and so the result is pleasing to the eye.

Assignment

Turn to Assignment Booklet 5A and complete the assignment for Section 1.

SECTION 2

Triangles and Trigonometry

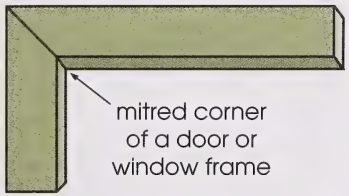
How tall is the derrick in the photograph? Can you find the height of this derrick without actually climbing it and having to run a tape measure from the top to the ground? You can if you use a method of measurement called indirect measurement.

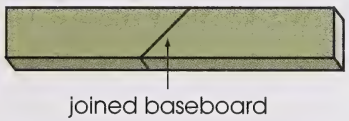
People like mathematicians, engineers, and navigators have been using indirect measurement of some kind for thousands of years. You had some experience with indirect measurement when you studied the Pythagorean Theorem in Section 1 of this module.

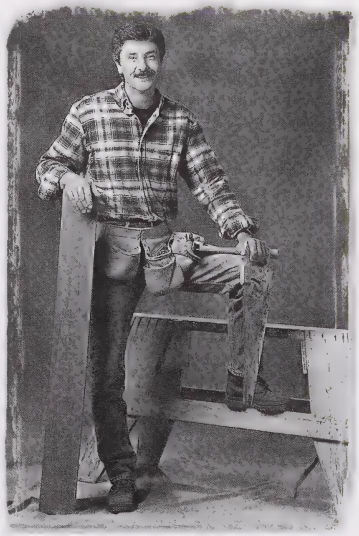
In this section, you will begin with some basic properties of congruent and similar triangles, which will lead to the development of the primary trigonometric ratios. Trigonometric ratios are the basis of the branch of mathematics called trigonometry. Trigonometry deals with the relationship between angles in a right triangle and the ratios of their sides. You will discover how to find the measures of missing angles and sides in a right triangle. You will then use your knowledge of trigonometric ratios to solve practical problems.

Activity 1: Classifying Angles

Knowledge of angles is important in many jobs in the everyday world. For example, carpenters need to know about angles when they mitre the corners of door or window frames or when they join baseboards.







How large must each of the angles be at the mitred corner of a door or window frame? To ensure the corner is square, what must these angles add up to? In mathematics, special names are given to related pairs of angles.

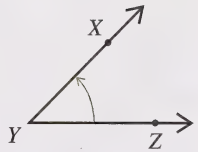
In this activity, you will examine and classify individual angles and pairs of angles.

Classifying Angles by Size

The following shows how to classify angles according to their size:

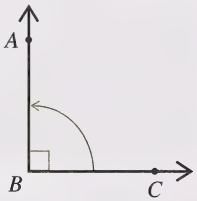
- If the size of an angle is less than a quarter turn, the angle is called an **acute angle**.

  For example, $\angle XYZ$ is an acute angle. It measures 45°.



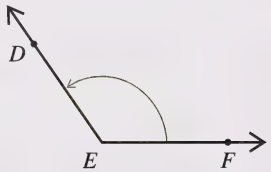
- If the size of an angle is exactly a quarter turn, the angle is called a **right angle**.

  For example, $\angle ABC$ is a right angle. It measures 90°.



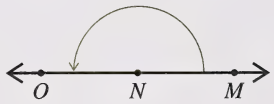
- If the size of an angle is more than a quarter turn and less than a half turn, it is called an **obtuse angle**.

  For example, $\angle DEF$ is an obtuse angle. It measures 125°.

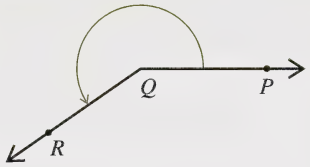


- If the size of an angle is exactly a half turn, the angle is called a **straight angle**.

  For example, $\angle MNO$ is a straight angle. It measures 180°.



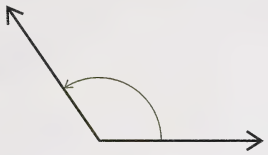
- If the size of an angle is more than half a turn and less than a full turn, it is called a **reflex angle**.



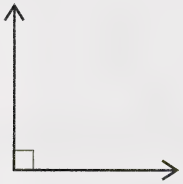
For example, $\angle PQR$ is a reflex angle. It measures 215°.

1. Without measuring, state whether each of the following is a right angle, acute angle, obtuse angle, straight angle, or reflex angle.

a.



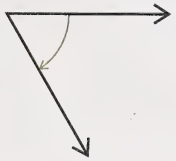
b.



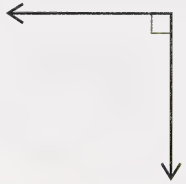
c.

d.

e.



f.

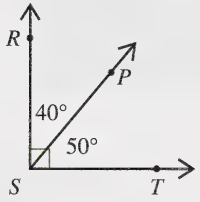


Check your answers by turning to the Appendix, page 102.

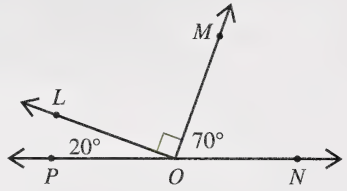
Classifying Pairs of Angles by Their Sum

The following shows how to classify pairs of angles by their sum:

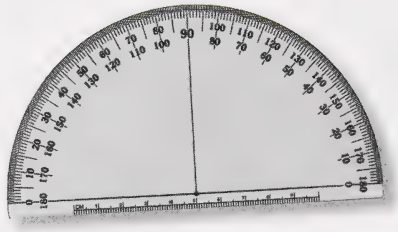
- Two angles that have a sum of 90° are **complementary angles**. For example, $\angle RSP$ and $\angle PST$ are complementary angles; $\angle LOP$ and $\angle MON$ are also complementary angles.



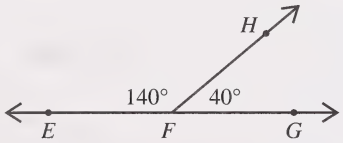
$40° + 50° = 90°$



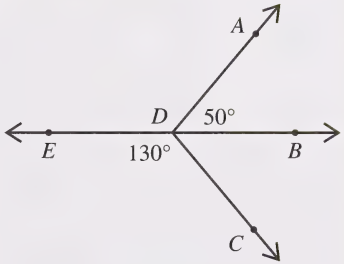
$20° + 70° = 90°$



- Two angles that have a sum of 180° are **supplementary angles**. For example, $\angle EFH$ and $\angle HFG$ are supplementary angles; $\angle ADB$ and $\angle EDC$ are also supplementary angles.



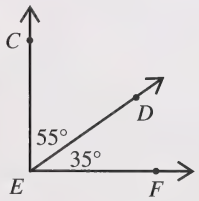
$140° + 40° = 180°$



$50° + 130° = 180°$

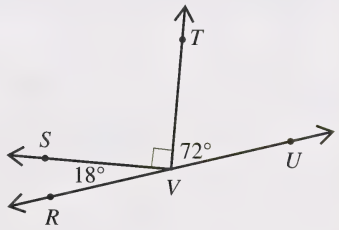
2. Use the given diagram to complete each of the following statements.

a.



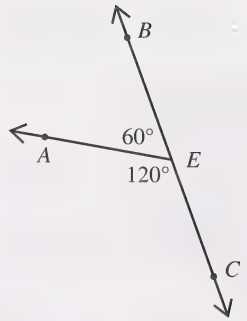
$\angle CED$ and $\angle DEF$ are _____ angles.

b.



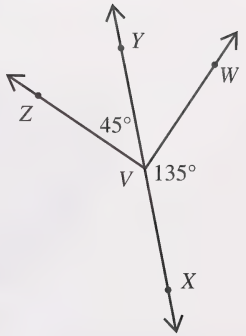
$\angle RVS$ and $\angle TVU$ are _____ angles.

c.



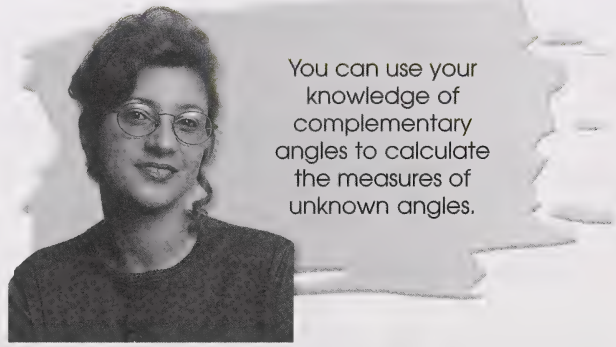
$\angle AEB$ and $\angle AEC$ are _____ angles.

d.



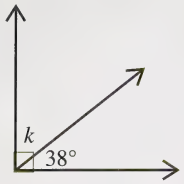
$\angle YVZ$ and $\angle WVX$ are _____ angles.

Check your answers by turning to the Appendix, page 102.



Example

Calculate the measure of the unknown angle, k.



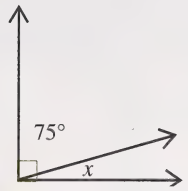
Solution

$k + 38° = 90°$ ← The sum of complementary angles is 90°.

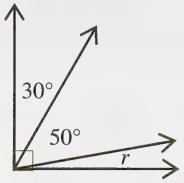
$k = 52°$

3. Calculate the measure of each of the unknown angles. Do **not** use a protractor.

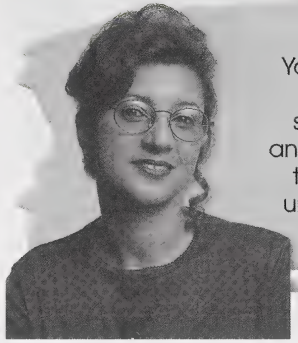
a.



b.



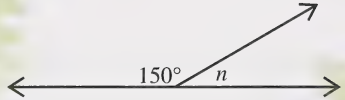
Check your answers by turning to the Appendix, page 102.



You can use your knowledge of supplementary angles to calculate the measure of unknown angles.

Example

Calculate the measure of the missing angle, n.



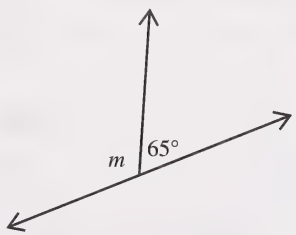
Solution

$n + 150° = 180°$ ← The sum of supplementary angles is 180°.

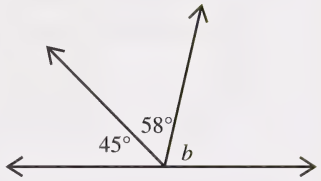
$n = 30°$

4. Calculate the measure of each of the unknown angles. Do **not** use a protractor.

a.



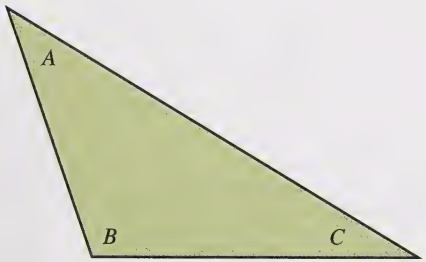
b.



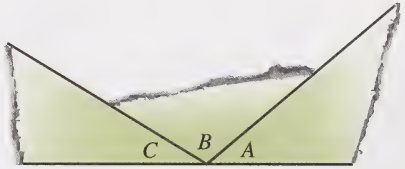
Check your answers by turning to the Appendix, page 102.

Angles are formed where the side of a figure meet. The angles inside the figure are called interior angles.

The following triangle has three interior angles. The angles are labelled *A*, *B*, and *C*.



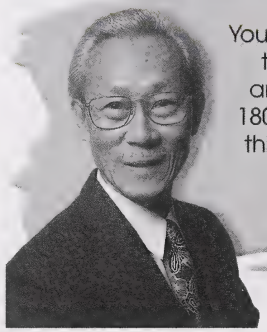
If you cut out the triangle in the example and carefully tear off the three corners, the pieces can be placed side by side.



Because the angles form a straight angle, you can conclude that the sum of the angles of ΔABC is 180°.

5. Draw several triangles that are of different sizes and shapes. Cut out each triangle and carefully tear off the three corners of each; then place the pieces side by side. Do the three angles of each triangle form a straight angle?

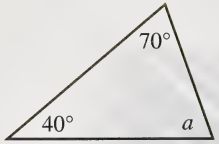
Check your answer by turning to the Appendix, page 102.



You have just discovered that the sum of the angles of a triangle is 180°. Now, you can use this knowledge to find the measures of unknown angles.

Example

Find the measure of the missing angle.

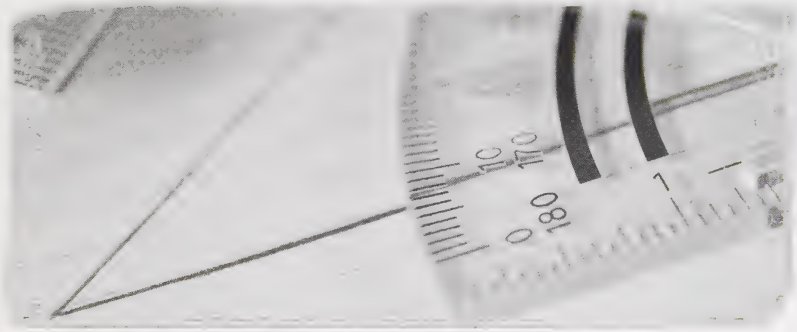


Solution

$70° + 40° + a = 180°$ ← The sum of the angles of a triangle is 180°.

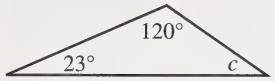
$110° + a = 180°$

$a = 70°$

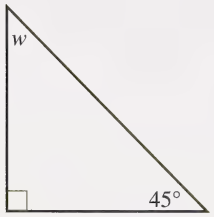


6. Calculate the measure of the missing angle.

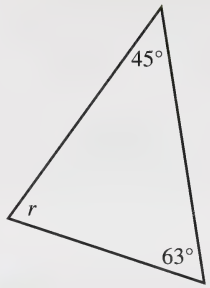
a.



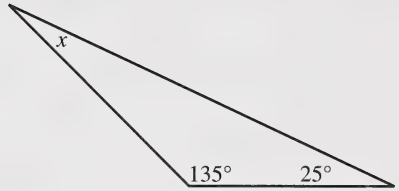
b.



c.

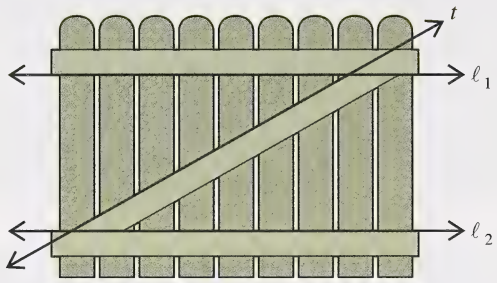


d.



Check your answers by turning to the Appendix, page 102.

Classifying Angles Formed by Parallel Lines and a Transversal



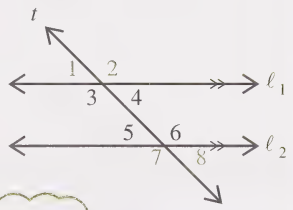
This fence suggests a pair of parallel lines cut by a transversal.

A transveral is a line intersecting two or more other lines.

When a pair of parallel lines is cut by a transversal, eight angles are formed. These angles can be classified according to their position.

- Angles that are on the outside of the parallel lines are **exterior angles**.

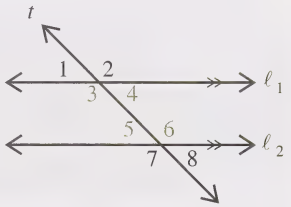
  For example, $\angle 1$, $\angle 2$, $\angle 7$, and $\angle 8$ are exterior angles.



The symbol ⇉ on both lines indicates that the lines are parallel.

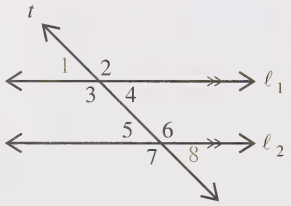
- Angles that are on the inside of the parallel lines are **interior angles**.

  For example, $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ are interior angles.



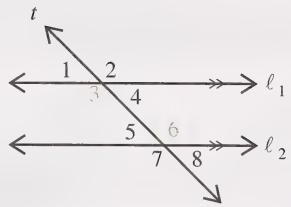
- Two exterior angles **not** on the same side of the transversal are called **exterior alternate angles**.

  For example, $\angle 1$ and $\angle 8$ are exterior alternate angles.

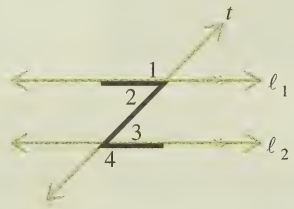


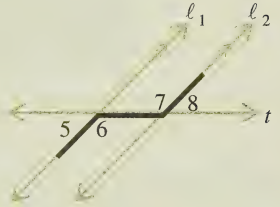
- Two interior angles **not** on the same side of the transversal are called **interior alternate angles**.

  For example, $\angle 3$ and $\angle 6$ are interior alternate angles.



Note: Pairs of interior alternate and exterior alternate angles of parallel lines form a letter Z. The angles in each pair are equal.





$\angle 1$ and $\angle 4$ are exterior alternate angles; $\angle 1 = \angle 4$.

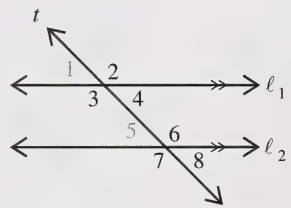
$\angle 2$ and $\angle 3$ are interior alternate angles; $\angle 2 = \angle 3$.

$\angle 5$ and $\angle 8$ are exterior alternate angles; $\angle 5 = \angle 8$.

$\angle 6$ and $\angle 7$ are interior alternate angles; $\angle 6 = \angle 7$.

- If an angle can be slid along the transversal onto a second angle, the pair of angles are called **corresponding angles**.

  For example, $\angle 1$ and $\angle 5$ are corresponding angles.



Note: Corresponding angles of parallel lines form a letter F. The angles in each pair are equal.

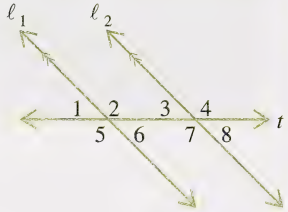
$\angle 1$ and $\angle 3$ are corresponding angles; $\angle 1 = \angle 3$.

$\angle 2$ and $\angle 4$ are corresponding angles; $\angle 2 = \angle 4$.

$\angle 5$ and $\angle 7$ are corresponding angles; $\angle 5 = \angle 7$.

$\angle 6$ and $\angle 8$ are corresponding angles; $\angle 6 = \angle 8$.

7. Make three diagrams similar to the one given. Use a coloured pen to draw the lines, and use a black pen to label the diagram.



a. Use a black pen to show the letter **F** on the first diagram. Identify the corresponding angles.

b. Use a black pen to show the letter **Z** on the second diagram. Identify the interior alternate and exterior alternate angles.

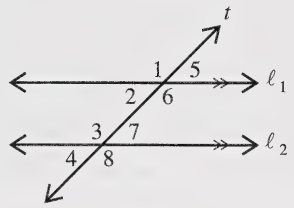
8. Use the diagram given to complete the following statements.

a. $\angle 5$ and $\angle 7$ are _____ angles.

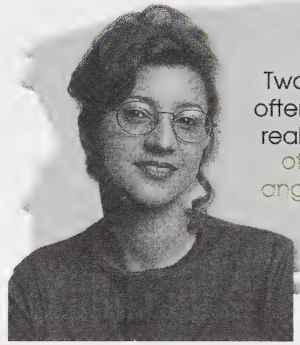
b. $\angle 5$ and $\angle 4$ are _____ angles.

c. $\angle 2$ and $\angle 7$ are _____ angles.

d. $\angle 2$ and $\angle 4$ are _____ angles.



Check your answers by turning to the Appendix, page 103.

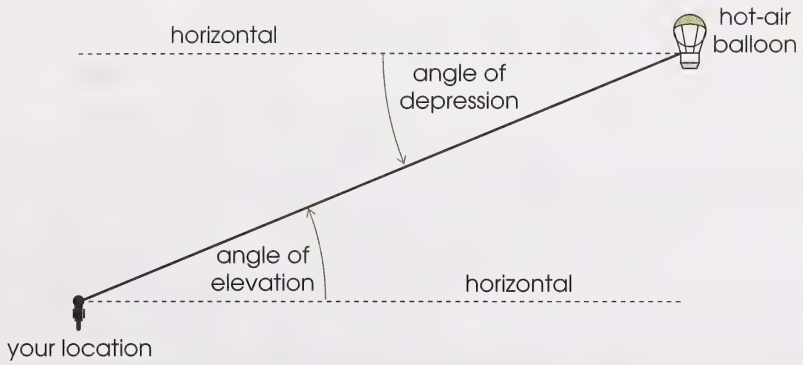


Two angles that are often discussed in the real world are angle of elevation and angle of depression.

The angle of elevation is the angle through which an observer looks upward from the horizontal to see an object above.

The angle of depression is the angle through which an observer looks downward from the horizontal to see an object below.

For example, if you were looking up at a hot-air balloon, the angle through which you would have to raise your eyes from the horizontal to see the balloon is the angle of elevation. A passenger in the balloon would have to look down from the horizontal to see you. This angle is the angle of depression.



9. Is the angle of depression equal to the angle of elevation? Use the preceding diagram to explain your answer.

Check your answers by turning to the Appendix, page 103.

Looking Back

In this activity, you classified angles by size, classified angles by their sum, and classified angles formed by parallel lines and a transversal. You also discovered that the sum of the angles of a triangle is 180°.

10. In your journal, complete the sentences using words from the following list of words. This list will provide you with a summary for studying later.

- 90°
- 180°
- straight
- right
- obtuse
- supplementary
- complementary
- reflex

The sum of the angles in a triangle is _____.

An acute angle is less than _____, a _____ angle is exactly 90°, and an _____ angle is between 90° and 180°.

A _____ angle is exactly 180° and an angle between 180° and 360° is a _____ angle.

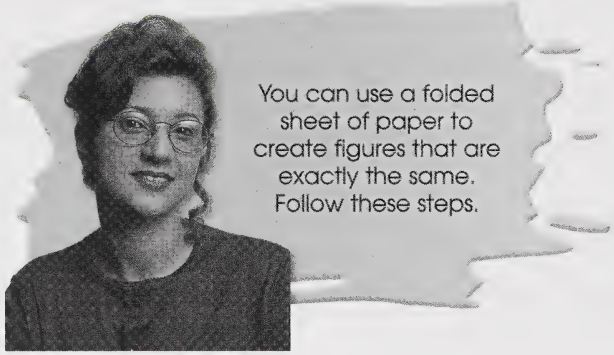
If two angles form a straight angle, they are _____ angles. If two angles form a right angle, they are _____ angles.

Check your answers by turning to the Appendix, page 103.

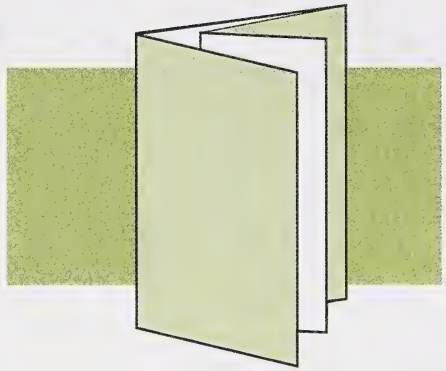
Activity 2: Congruent Triangles

In architectural design, triangles are often repeated to create pleasing shapes as well as strong structures. Can you find triangles that appear to be exactly the same in the photograph?

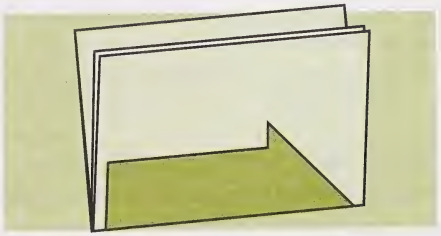
In this activity, you will analyse triangles that are the same size and shape and discover the minimum information you need to know before you can decide that two triangles are congruent.



Step 1: Fold a sheet of paper in half. Then fold the result in half again (as shown in the diagram).



Step 2: Draw half a figure (such as an arrow along the second folded edge of the paper), and cut out the figure.



1. Compare the two figures. Do they coincide exactly when you place one on the other?

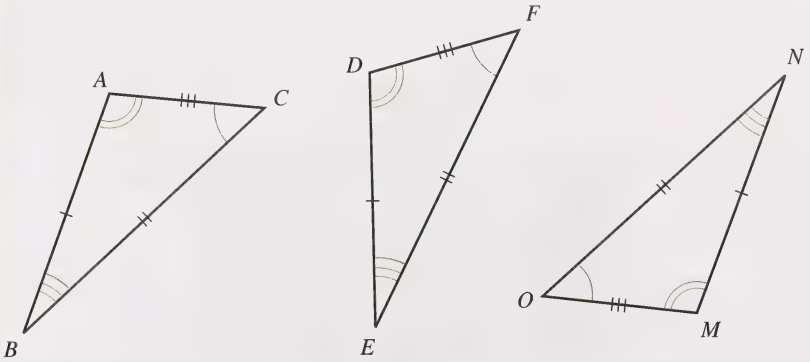
2. Do the two figures appear to be the same size and shape?

3. Suppose you were to fold the paper three times along its length. How many figures would you get? Do all these figures appear to be the same size and shape?

Check your answers by turning to the Appendix, page 104.

Figures that are identical in size and shape are called congruent figures.

All of the following triangles have the same size and shape.



Because the preceding triangles are the same size and shape, you can say they are congruent. That is, $\Delta ABC \cong \Delta DEF \cong \Delta MNO$. **Note:** The vertices of the triangles are always listed in the order of their corresponding angles.

The symbol $\cong$ means "is congruent to."

$\Delta ABC \cong \Delta DEF \cong \Delta MNO$ means that the pairs of corresponding angles and corresponding sides have equal measurements.

Angles	Sides
$\angle A = \angle D = \angle M$	$AB = DE = MN$
$\angle B = \angle E = \angle N$	$BC = EF = NO$
$\angle C = \angle F = \angle O$	$AC = DF = MO$

These six statements can always be made about any two congruent triangles.

It is not necessary to know that all of the pairs of corresponding sides and corresponding angles are equal to show that two triangles are congruent. Now, you will see if you can discover how much information is necessary to show or draw two or more congruent triangles.

4. Use a ruler, protractor, and compass to accurately draw three triangles for each of the following properties. Label each triangle with the property number; then cut them out. You will end up with 15 triangles.

Property 1: The sides are 4 cm, 6 cm, and 8 cm in length.

Property 2: The angles are 40°, 60°, and 80°.

Property 3: Two angles are 50° and 60° and the side joining these two angles is 7 cm.

Property 4: One angle is 80° and the two sides that include the given angle are 4 cm and 6 cm long.

Property 5: Two sides are 6 cm and 5 cm and one of the angles not formed by the given sides is 50°.

5. Look at the triangles in each group from question 4 according to their property numbers. State whether all possible triangles in each group are congruent.

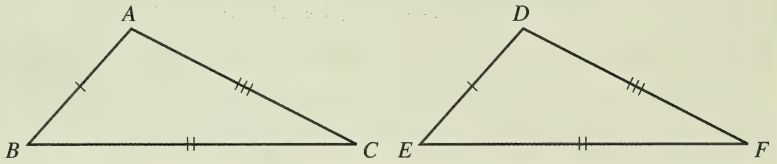
6. Based on your observations, how much information is necessary to draw two or more congruent triangles?

Check your answers by turning to the Appendix, page 104.

There are three necessary conditions under which two triangles are congruent.

Condition 1: SSS (side, side, side)

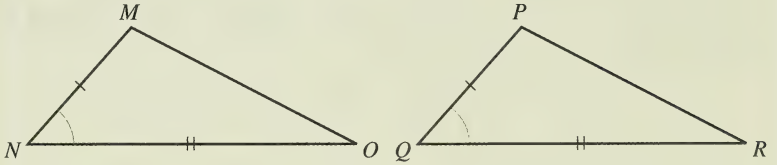
All three sides of one triangle are equal to the corresponding sides of the other triangle.



$\therefore \Delta ABC \cong \Delta DEF$

Condition 2: SAS (side, angle, side)

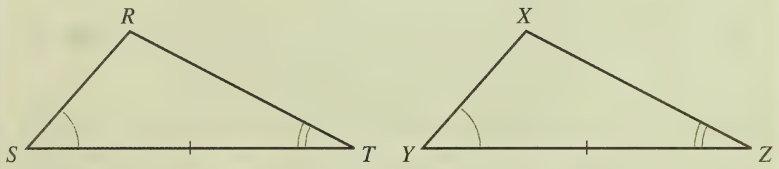
Two sides and the contained angle of one triangle are equal to the corresponding two sides and the contained angle of the other triangle.



$\therefore \Delta MNO \cong \Delta PQR$

Condition 3: ASA (angle, side, angle)

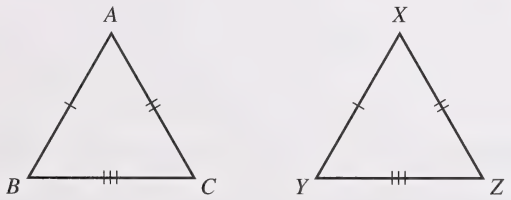
Two angles and a side of one triangle are equal to the corresponding two angles and side of the other triangle.



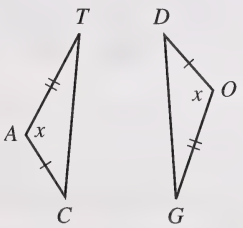
$\therefore \Delta RST \cong \Delta XYZ$

7. The following pairs of triangles are congruent. Which conguence condition is used to assert that the two triangles are congruent? Write the congruence statement for each pair of triangles.

a.



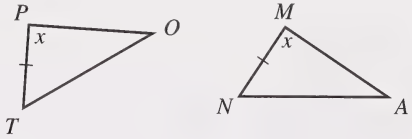
b.



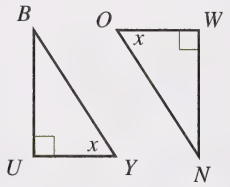
8. List all the corresponding congruent parts of the congruent triangles in question 7.

9. Name the missing congruent angles or congruent sides needed before you could state that the following pairs of triangles are congruent.

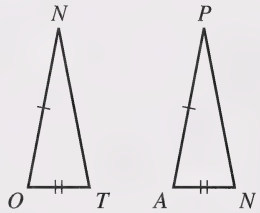
a.



b.

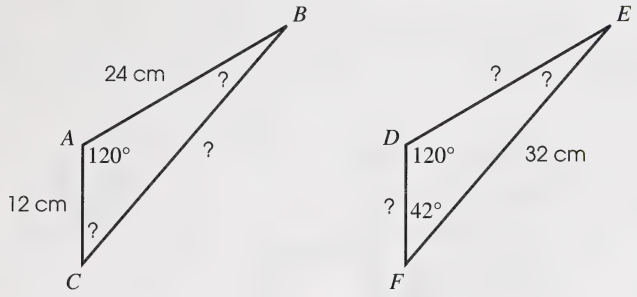


c.

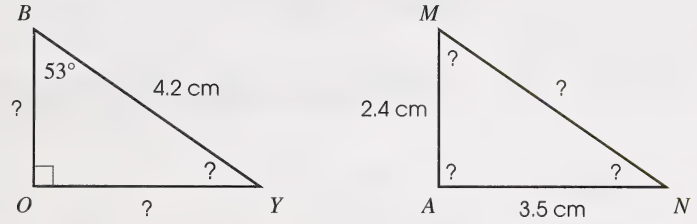


10. Given that the following pairs of triangles are congruent, find the missing measures (indicated by the ?).

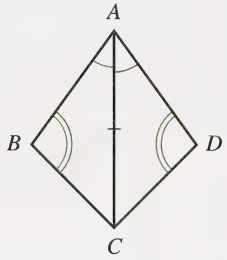
a. $\Delta ABC \cong \Delta DEF$



b. $\Delta BOY \cong \Delta MAN$



11. Use the diagram on the right to answer the following questions.



a. Why does $\angle ACB = \angle ACD$?

b. Are the two triangles congruent? Explain.

12. a. Are two triangles with the same perimeter always congruent, sometimes congruent, or never congruent?

b. Are two triangles with the same area always congruent, sometimes congruent, or never congruent?

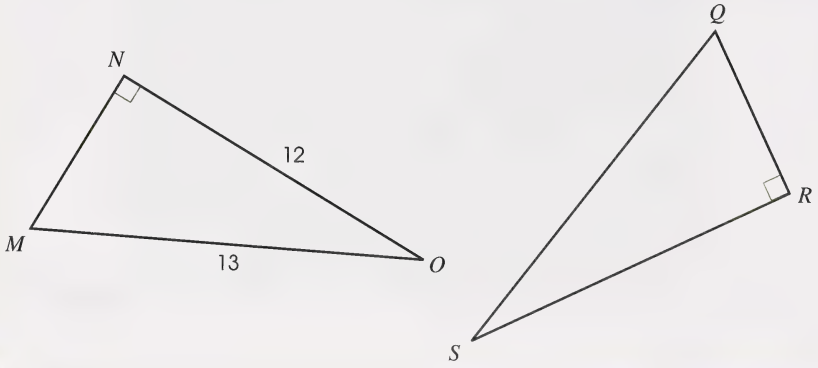
c. Are two squares with the same perimeter always congruent? Are two squares with the same area always congruent? Why is this true?

Check your answers by turning to the Appendix, page 105.

Looking Back

In this activity, you discovered that congruent triangles are the same size and shape. There are three conditions that ensure two triangles are congruent: SSS (side, side, side), SAS (side, angle, side), and ASA (angle, side, angle).

13. In your journal, copy the following triangles.



a. Calculate the length of side MN.

b. If $SR = 12$ and $QS = 13$, can you conclude the two triangles are congruent? Explain.

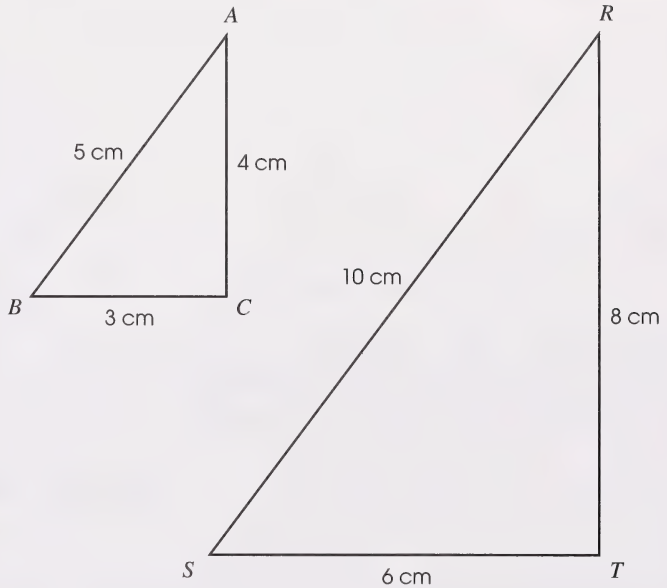
Check your answers by turning to the Appendix, page 106.

Activity 3: Similar Triangles

Do the two logos have the same shape and size? In Activity 1, you discovered that figures with the same shape and size are called congruent figures. In this activity, you will study figures that have the same shape but not the same size. These are called **similar figures**.

Triangles can be similar as well. Just as with congruent triangles, there are certain conditions that are necessary for similar triangles. In the following exercises, you will discover the conditions that are necessary for similar triangles.

Use the following two triangles to answer questions 1 and 2.



1. Measure the angles of ΔABC and ΔRST. Are the corresponding angles equal?

2. **a.** Find the following pairs of ratios.

i. $\frac{AC}{AB}, \frac{RT}{RS}$ **ii.** $\frac{BC}{AB}, \frac{ST}{RS}$ **iii.** $\frac{BC}{AC}, \frac{ST}{RT}$

b. What can you conclude about the ratio of the lengths of any two sides of ΔABC compared to the ratio of the lengths of the two corresponding sides of ΔRST?

3. Use the following two triangles to answer questions 3.a. and 3.b.

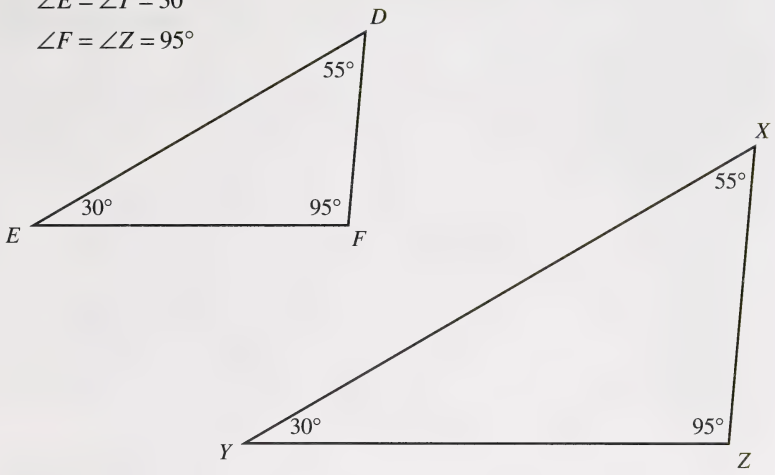
$\Delta DEF \sim \Delta XYZ$

The symbol ~ means "is similar to."

$\angle D = \angle X = 55°$

$\angle E = \angle Y = 30°$

$\angle F = \angle Z = 95°$



a. Measure the corresponding sides of ΔDEF and ΔXYZ, and write them as ratios in simplest form. Are these ratios equal?

b. Find the following pairs of ratios.

i. $\frac{DE}{DF}, \frac{XY}{XZ}$

ii. $\frac{DF}{EF}, \frac{XZ}{YZ}$

4. What do you notice about the ratios in question 3.b.? Explain.

5. In ΔABC, $\angle A = 40°$ and $\angle B = 60°$. In ΔDEF, $\angle D = 40°$ and $\angle E = 60°$.

a. What are the measures of $\angle C$ and $\angle F$? Explain.

b. Is $\Delta ABC \sim \Delta DEF$? Explain.

Check your answers by turning to the Appendix, page 106.

Properties of Similar Triangles

- If two triangles are similar, the corresponding angles are congruent and the corresponding sides are proportional.
- If corresponding angles are congruent, the two triangles are similar and the corresponding sides are proportional.
- If the corresponding sides of two triangles are proportional (have the same ratio), the triangles are similar and the corresponding angles are congruent.

These facts about similar triangles are useful when calculating distances without measuring them directly. Surveyors constantly make use of these techniques when performing their jobs.

Example

A surveyor wants to find the distance across a lake, but is unable to measure this distance directly. How can the surveyor determine the distance?

Solution

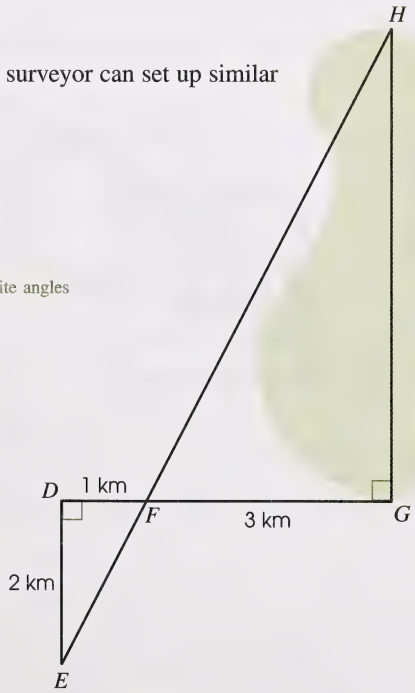
Using surveying equipment, the surveyor can set up similar triangles.

$\Delta FGH \sim \Delta FDE$

$\angle D = \angle G = 90°$

$\angle DFE = \angle GFH$ ← opposite angles

$\therefore \angle E = \angle H$



Because the two triangles are similar, the ratios of corresponding sides are equal.

$$\frac{DE}{GH} = \frac{DF}{GF}$$

$$\frac{2}{GH} = \frac{1}{3}$$

$$\cancel{GH}\left(\frac{2}{\cancel{GH}}\right) = GH\left(\frac{1}{3}\right)$$

$$2 = \frac{GH}{3}$$

$$3(2) = \cancel{3}\left(\frac{GH}{\cancel{3}}\right)$$

$$6 = GH$$

$$GH = 6$$

The distance across this lake is 6 km.

Example

Find the height of the flagpole.

Solution

$\Delta ABC \sim \Delta EBD$

Therefore, the corresponding sides have the same ratio.

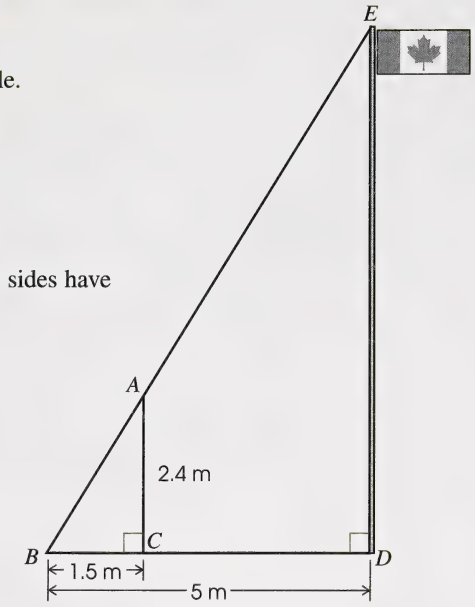
$$\frac{DE}{AC} = \frac{BD}{BC}$$

$$\frac{DE}{2.4} = \frac{5}{1.5}$$

$$2.4\left(\frac{DE}{2.4}\right) = 2.4\left(\frac{5}{1.5}\right)$$

$$DE = \frac{12}{1.5}$$

$$= 8$$



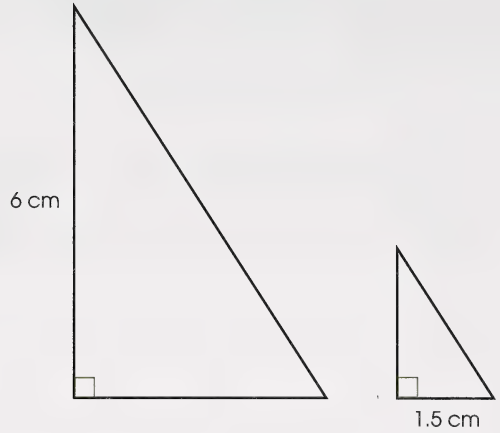
Note: When setting up the ratio, place the unknown in the numerator if possible. This will reduce the number of steps in the calculation.

The flagpole is 8 m tall.

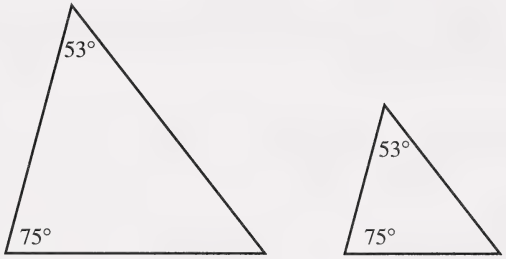
6. Determine whether the triangles in each pair are similar. Use the measurements given about the triangles to help you determine your answer. Explain.

a.

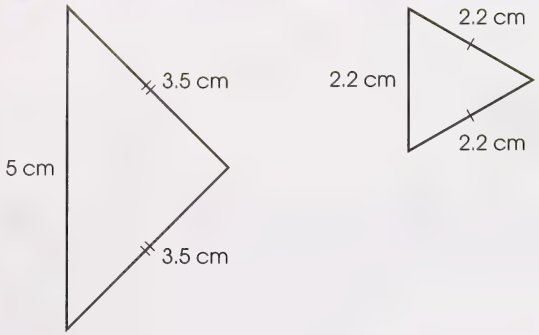
b.



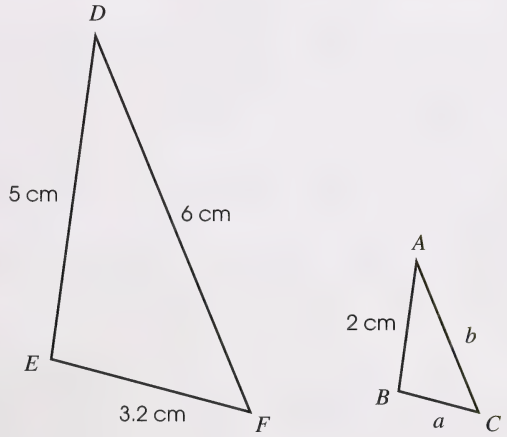
c.



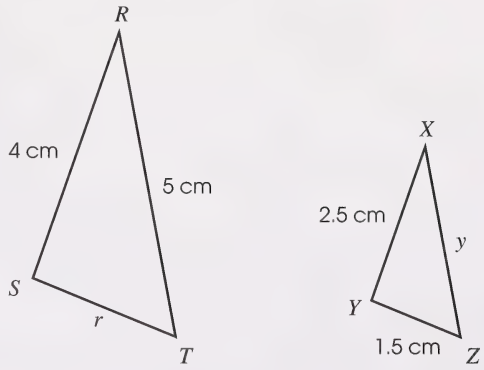
d.



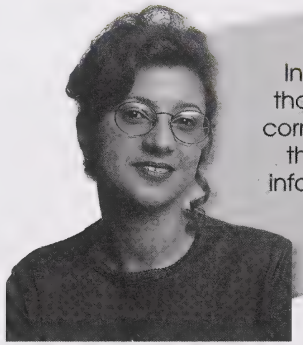
7. a. ΔABC and ΔDEF are similar. Find a and b.



b. ΔRST and ΔXYZ are similar. Find r and y.



Check your answers by turning to the Appendix, page 107.

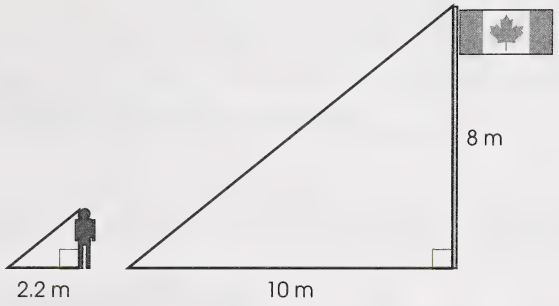


In Activity 1, you discovered that congruent triangles have corresponding angles and sides that are congruent. Use this information to help answer the following questions.

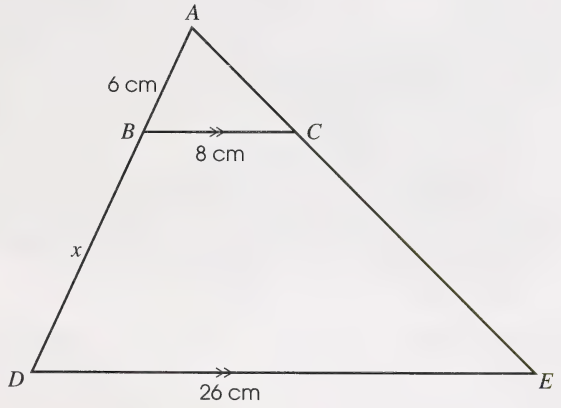
8. Are congruent triangles always similar? Explain.

9. Are similar triangles always congruent? Explain.

10. An 8-m flagpole casts a 10-m shadow. At the same time of day, Joanna casts a 2.2-m shadow. How tall is Joanna?

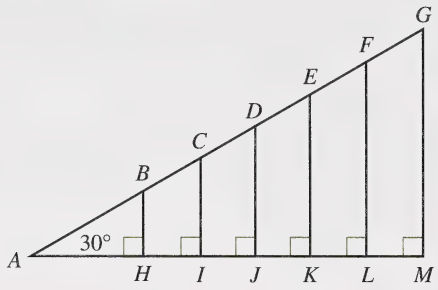


11. Find the measure of x.



12. Are all squares similar? Explain.

13. The diagram shows a set of right triangles. Use the diagram to answer the questions 13.a. to 13.g.



a. Why is $\Delta ABH \sim \Delta ACI \sim \Delta ADJ$?

b. Which other triangles are similar to ΔABH ?

c. Measure BH and AB. What is the ratio $\frac{BH}{AB}$?

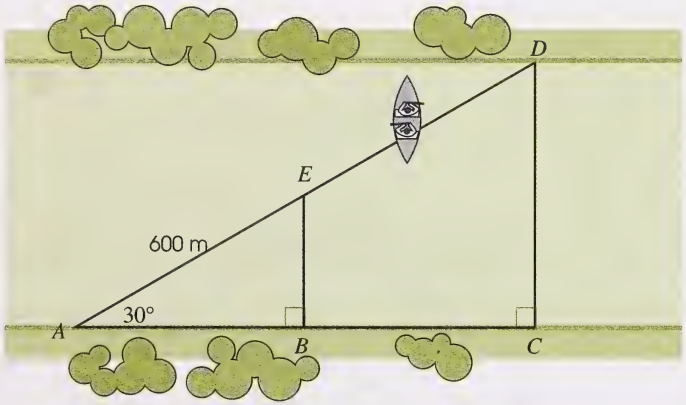
d. Measure CI and AC. What is the ratio $\frac{CI}{AC}$?

e. Predict what the ratios of $\frac{DJ}{AD}$, $\frac{EK}{AE}$, $\frac{FL}{AF}$, and $\frac{GM}{AG}$ are going to be. Explain.

f. If you extended AG to X and AM to Y so that $XY \perp AY$ and $XY = 9.5$ cm, what is the measure of AX?

g. What conclusion can you draw about a right triangle with a $30°$ angle?

14. Fredina and Maurice are canoeing across the North Saskatchewan River. The current is strong; so, they are unable to paddle straight across. In fact, their path makes a 30° angle with the riverbank they are paddling from.



a. When they are halfway across the river, they have travelled 600 m. How far from the riverbank are they?

b. How wide is the river where they are crossing?

c. If Fredina and Maurice wanted to land at a certain spot on the opposite riverbank, about how far upstream should they begin their crossing? Round your answer to the nearest metre.

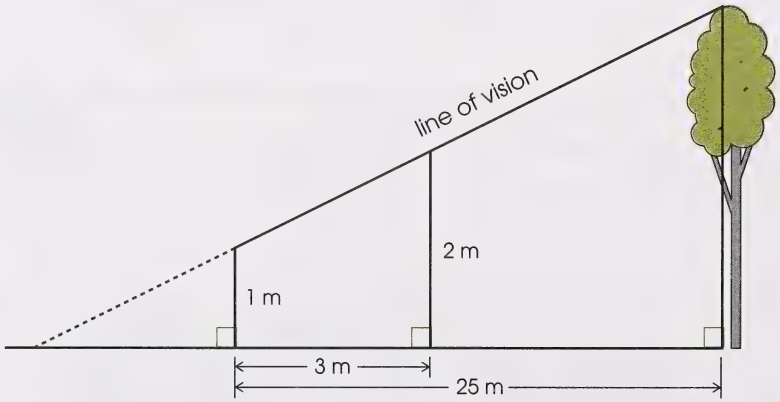
Check your answers by turning to the Appendix, page 108.

Looking Back

In this activity, you discovered the conditions necessary for similar triangles and solved problems involving similar triangles.

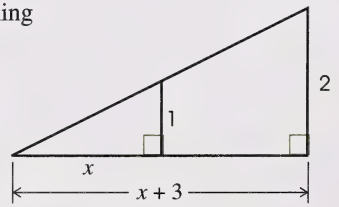
15. In your journal, copy the following problem involving similar triangles.

An estimate of the height of a tree is required before it is chopped down. Darren kneels 3 m from a fence that is 2 m high. Darren is 25 m from the tree and can just see the top of the tree over the fence when his eyes are 1 m above ground level.



a. Begin solving the problem by determing the value of x. Solve the following equation:

$$\frac{x}{1} = \frac{x+3}{2}$$

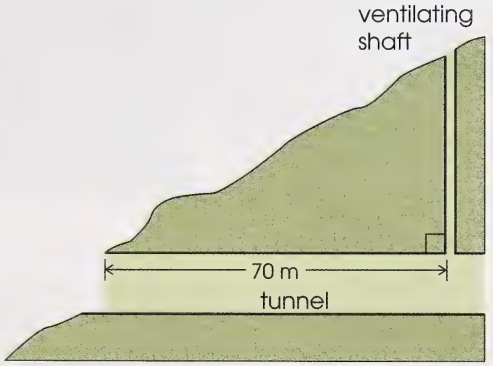


b. Determine an approximate height of the tree. Round your answer to the nearest tenth.

Check your answers by turning to the Appendix, page 110.

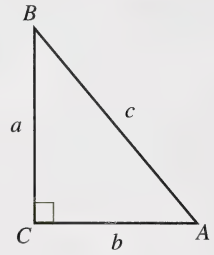
Activity 4: Trigonometry

A horizontal tunnel is being excavated through a hill. The first vertical ventilating shaft must be drilled no more than 70 m from the entrance of the tunnel. The engineers involved in this project need to find the height of this shaft and how far up the hill the shaft will exit.



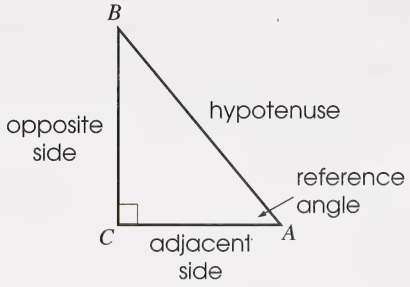
To answer these questions, the engineers need to use a branch of mathematics called trigonometry. Trigonometry involves the study of the relationships of angles and sides of right triangles.

To discuss and clearly understand trigonometry, you must learn some basic vocabulary first. As you may recall from your previous work with the Pythagorean Theorem, each side of a triangle can be named by using the lower-case letter that corresponds to the letter representing the opposite angle.



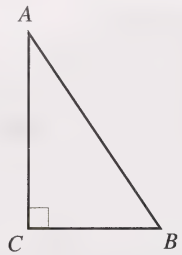
A right triangle has one right angle and two acute angles. The side opposite the right angle is the **hypotenuse**. The legs of the triangle have names as well, but they are named according to the position of the reference angle.

The leg directly opposite the reference angle is called the **opposite side**, and the leg next to the reference angle is called the **adjacent side**.

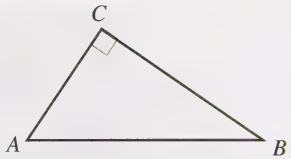


1. Name the opposite side, adjacent side, and hypotenuse with reference to $\angle A$ in each of the following right triangles.

a.

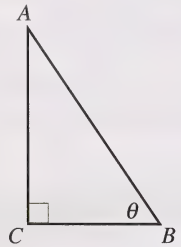


b.

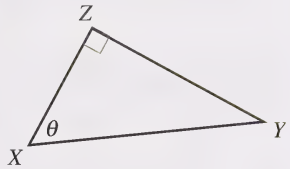


2. Name the sides appropriately with reference to $\angle\theta$ in each of the following right triangles.

a.



b.



Greek letters, such as θ and α, are sometimes used to represent angles.

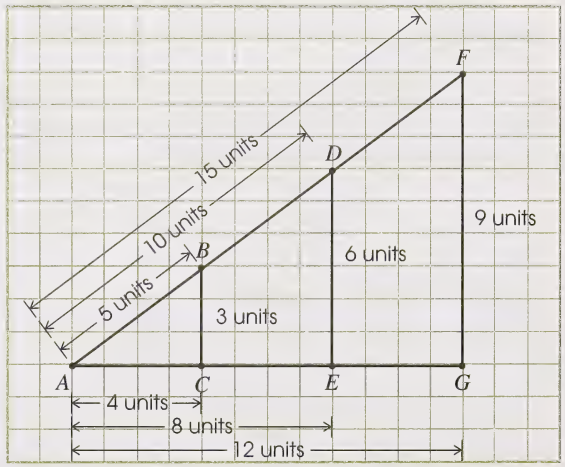
Check your answers by turning to the Appendix, page 110.

You will now use the properties of similar triangles, which you explored in Activity 2, to develop the trigonometric ratios.

Tangent Ratio

Obtain a sheet of grid paper and draw a right triangle with a base length of 4 units and a height of 3 units. Extend the triangle to make one with a base length of 8 units and a height of 6 units and another with a base length of 12 units and a height of 9 units.

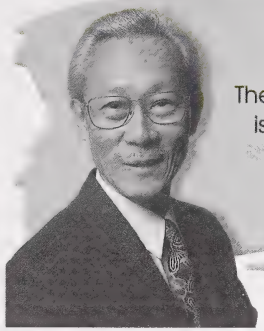
Your drawing should look like the following.



3. a. Using $\angle A$ as the reference angle, find the ratio of the opposite side, BC, to the adjacent side, AC. Do the same for ΔADE and ΔAFG. Find the decimal value for the ratio for each triangle.

b. Is the ratio in question 3.a. the same for each triangle?

Check your answers by turning to the Appendix, page 111.

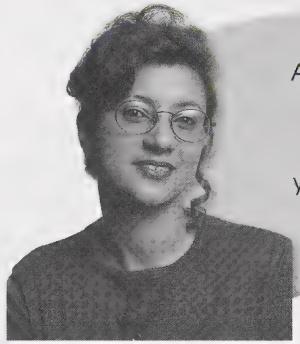


The ratio in question 3.a. is called the tangent ratio for $\angle A$.

If $\angle A$ is an acute angle in a right triangle, $\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent } \angle A}$. Often, you will see the tangent ratio for a given angle, A, written in short form as $\tan A = \frac{\text{opp}}{\text{adj}}$.

4. Use a protractor to find the measure of $\angle A$ in the diagram preceding question 3. Round your answer to the nearest degree.

Check your answers by turning to the Appendix, page 111.



Angles can be measured in degrees or radians. In this module, you will only use degrees. So, make sure your calculator is in degree mode (deg).

You can use a scientific calculator to find the ratio of an angle expressed as a decimal. Perform the following keystroke sequence.

Note: Some calculators may require you to enter the number first. Consult your manual for order of operation.

5. What value appears in the display? Round this value to 4 decimal places.

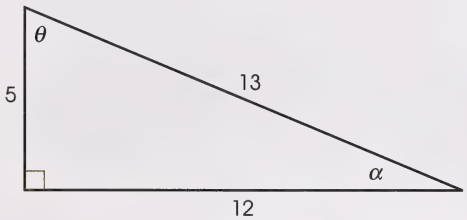
6. How does this value compare to the tangent ratio you obtained previously?

7. You can use a scientific calculator to find the tangent ratio of any angle. Find the tangent ratio of the following angles using a scientific calculator. Round your answers to 4 decimal places where necessary.

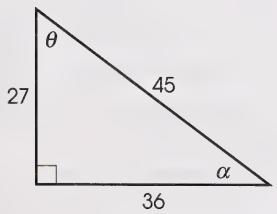
   a. 81° **b.** 22° **c.** 45° **d.** 2°

8. Using the values provided, find the tangent ratios for θ and α. Express your answers as fractions in simplest form and as decimals. Round your answers to 4 decimal places where necessary.

a.



b.



Check your answers by turning to the Appendix, page 111.

You can also use a scientific calculator to find the angle if you are given the tangent ratio.

When you obtain the tangent of an angle using your calculator, you know that you get the ratio of the opposite side over the adjacent side expressed as a decimal. If you are given this ratio, you can use the inverse of the tangent function to obtain the angle. Press [2nd] [tan].

Example

If $\tan A = 0.5000$, find the measure $\angle A$. Round your answer to the nearest degree.

Solution

To find the measure of $\angle A$, enter the following keystrokes.

[2nd] [tan] [•] [5] [=]

$\therefore \angle A \doteq 27°$

Note: Your calculator may require you to use [IN] [tan] or [Shift] [tan]. Consult your manual.

Use a scientifc calculator to answer questions 9 and 10. Make sure your calculator is in degree mode.

9. Find the angle from the following tangent ratios. Round your answers to the nearest degree.

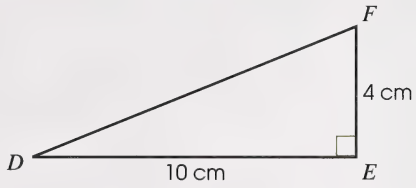
a. $\tan\theta = 0.3985$

b. $\tan\theta = 9.8476$

c. $\tan\theta = \frac{5}{7}$

d. $\tan\theta = \frac{4}{3}$

10. Find the tangent ratio and the measure of each acute angle in ΔDEF.

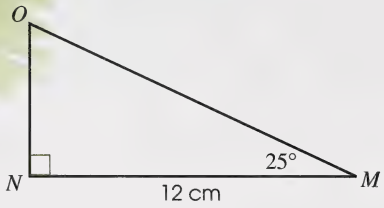


Check your answers by turning to the Appendix, page 112.

You can use the tangent ratio to solve for the length of a side in a right triangle if you know one acute angle and one of the two shorter sides.

Example

In ΔMNO, find the length of NO. Round your answer to the nearest tenth.



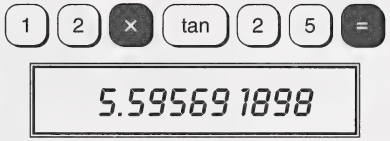
Solution

$$\tan M = \frac{\text{opp}}{\text{adj}}$$

$$\tan 25° = \frac{NO}{12}$$

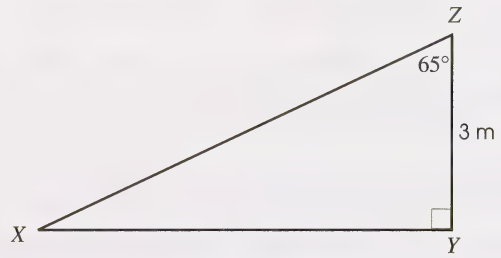
$$12(\tan 25°) = 12\left(\frac{NO}{12}\right)$$

$$NO = 12 \tan 25°$$



The length of NO is about 5.6 cm.

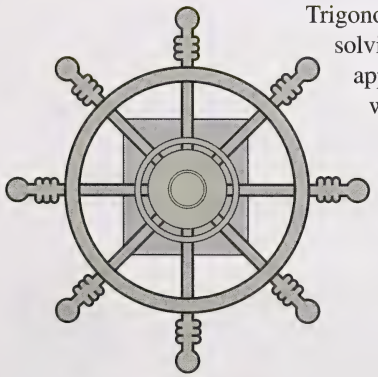
11. Use the information in ΔXYZ to answer the following.



a. Calculate the length of XY. Round your answer to the nearest tenth of a metre.

b. Calculate $\angle X$ using two methods.

Check your answers by turning to the Appendix, page 113.



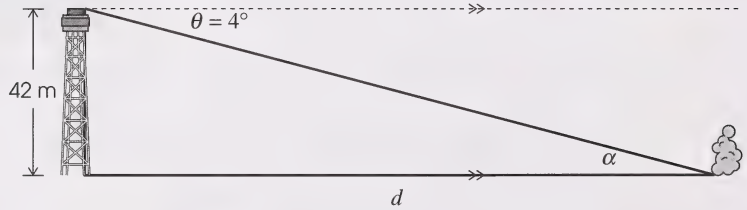
Trigonometry was initially developed for use in solving problems in navigation. Since then, its application has been broadened to include a wide range of problems in mathematics, science, and industry. For example, the tangent ratio is used with many problems involving heights and distances.

Example

Colleen, a forest ranger, is standing at the top of a lookout tower. She notices some smoke rising from the forest. Using an instrument called a transit, Colleen finds the angle of depression for the source of the smoke to be 4°. If the tower is 42 m high, how far from the lookout tower is the source of the smoke? Round your answer to the nearest metre.

Solution

Step 1: Draw a diagram of the situation.



Step 2: Determine α.

Because the horizontal lines are parallel, θ and α are equal.

$\therefore\ \alpha = 4°$

Step 3: Use the correct trigonometric ratio to solve for d, the distance to the smoke.

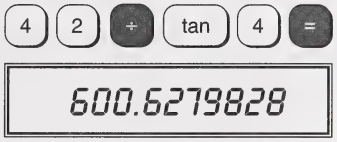
$$\tan 4° = \frac{42}{d}$$

$$d(\tan 4°) = \cancel{d}\left(\frac{42}{\cancel{d}}\right)$$

$$d \times \tan 4° = 42$$

$$\frac{d \times \tan 4°}{\tan 4°} = \frac{42}{\tan 4°}$$

$$d = \frac{42}{\tan 4°}$$

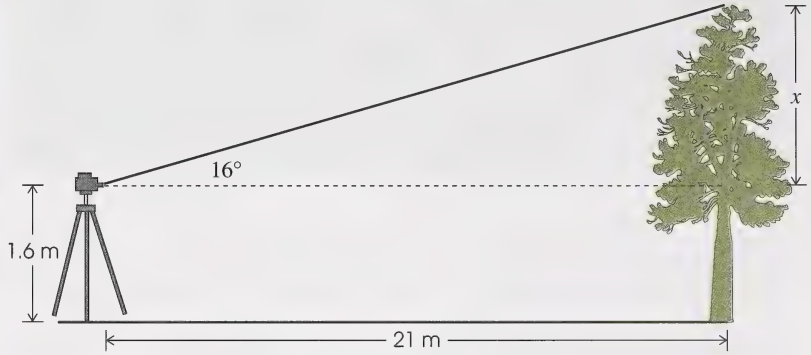


The smoke is about 601 m from the base of the lookout tower.

12. A tree casts a shadow 38 m long when the sun's rays are at an angle of 42° to the ground. Find the height of the tree to the nearest metre.

13. A sailboat is sighted from a lighthouse situated 125 m above the level of the water. If the angle of depression from the lighthouse to the highest tip of the sailboat is 6° and the tip of the sailboat is 10 m above the level of the water, how far is the sailboat from shore? Round your answer to the nearest metre.

14. Calculate the height of the tree to the nearest tenth of a metre using the indicated measurements.



Check your answers by turning to the Appendix, page 114.

There are many types of instruments used for determining distances and measuring angles of elevation and angles of depression. A clinometer is an instrument used for measuring angles of elevation and depression. You can even make your own clinometer.

Constructing and Using a Clinometer

Materials

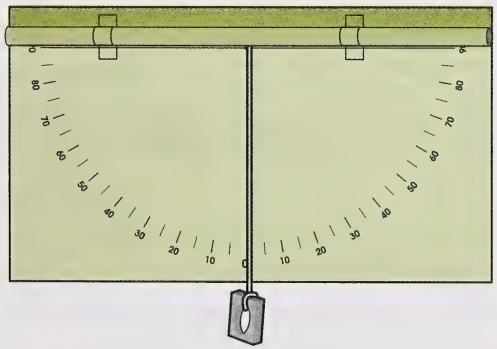
- a drinking straw or pencil
- a piece of string or thread 20 cm long
- a weight (such as a nut)
- adhesive tape and glue
- the cut-out located at the back of the Appendix
- a piece of cardboard

Construction

Step 1: Cut out the clinometer located at the back of the Appendix and glue it on a piece of cardboard of the same size.

Step 2: Tape the drinking straw where indicated. Make sure the straw is parallel to the 90° line.

Step 3: Tie the nut to the string, and thread the string through the centre point so it hangs about 4 cm below the cardboard and through 0°. Tape the string securely to the back with adhesive tape.



How To Use a Clinometer

Step 1: Point the straw at the top of the object you wish to measure. Look through the straw and make sure it is sighted as accurately as possible.

Step 2: Have another person read the angle of the string. If you have to do it yourself, hold the string with your thumb once you have it properly aligned; then read the angle afterward. **Note:** You will have to estimate to the nearest degree because the angles have only been marked at every 5°.

Practice

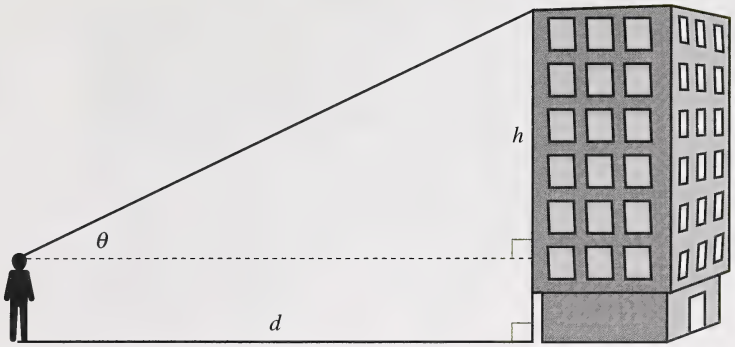
Practise using your clinometer by finding the height of a nearby building.

Step 1: Stand with your clinometer where you can see the top of the building.

Step 2: Use your clinometer to measure the angle of elevation.

Step 3: Measure the distance, d, from where you are standing to the base of the building.

Step 4: Draw a diagram of your findings.



Step 5: Set up your equation using the correct trigonometric ratio.

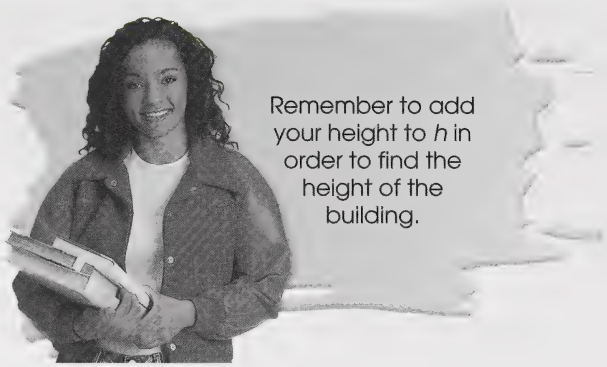
$$\tan\theta = \frac{h}{d}$$

Step 6: Rearrange the equation; substitute the values for d and θ; and solve for h.

$$d(\tan\theta) = \cancel{d}\left(\frac{h}{\cancel{d}}\right)$$

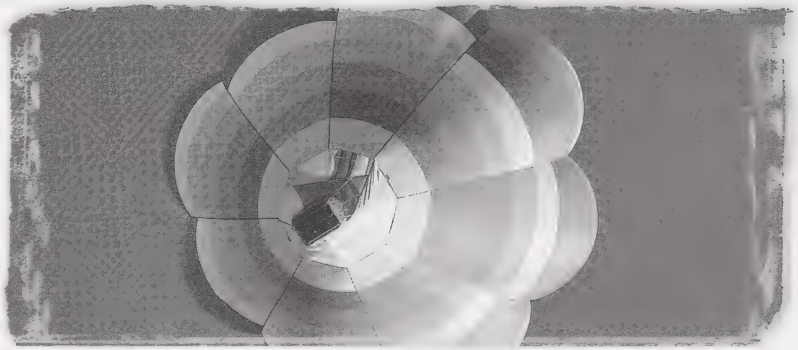
$$d \times \tan\theta = h$$

$$h = d \times \tan\theta$$

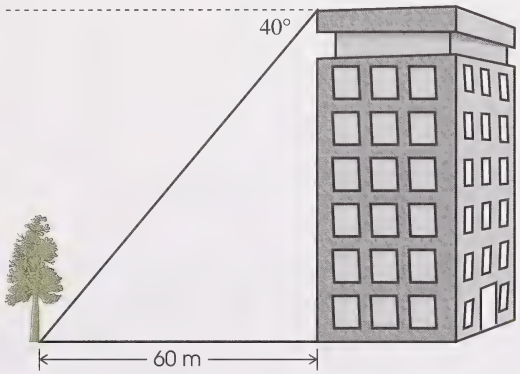


15. Use the clinometer to find the height of some building or object that is easy for you to find the actual height. Do you think your result is reasonable?

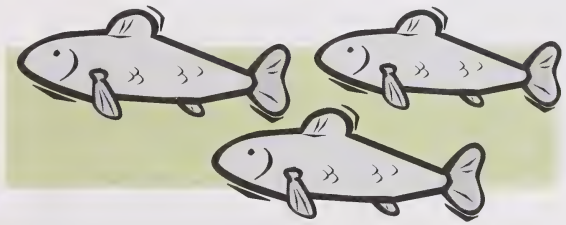
16. Sharon used a clinometer and a tape measure to find the approximate height of a hot-air balloon. The angle of elevation was 20° and the distance to a point on the ground below the balloon was 80 m. What did she find the height of the inflated hot-air balloon to be if her height is 1.7 m? Round your answer to the nearest tenth of a metre.



17. From the top of a building, the angle of depression to the base of a tree 60 m away is 40° (as measured by a clinometer). Find the height of the building to 1 decimal place. **Remember:** The angle of depression is equal to the angle of elevation.



18. By using sonar, the captain of a fishing schooner detects a school of fish at a depth of 58.4 m. If the angle of depression is 16°, how far must the captain move the schooner in order to be directly over the school of fish? Round your answer to the nearest metre.



19. a. Use the tangent key on your calculator to complete the following table. Round the tangent values to 4 decimal places before writing them in the table.

Angle	Tangent	Angle	Tangent
5°		50°	
10°		55°	
15°		60°	
20°		65°	
25°		70°	
30°		75°	
35°		80°	
40°		85°	
45°		90°	

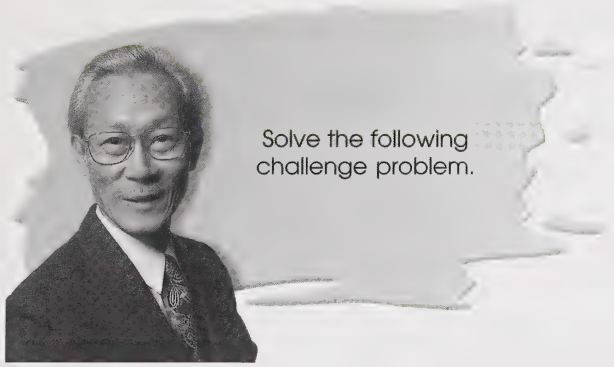
b. What pattern do you notice in the value of the tangent ratio as the angle increases from 5° to 85°?

c. What do you think the tangent value for 0° will be? Test your prediction with your calculator.

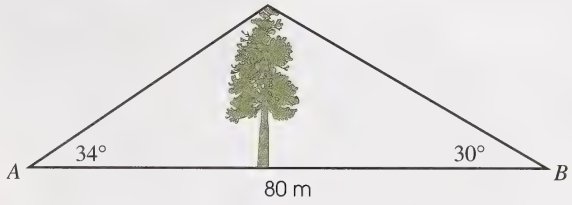
d. Why do you get an error message when you calculate the tangent value for 90°?

Check your answers by turning to the Appendix, page 115.

Now Try This



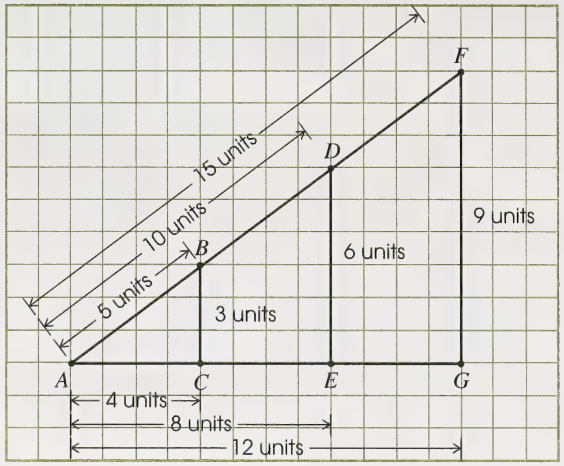
20. Using the following diagram, calculate the height of the tree to the nearest metre. (**Hint:** Let the distance from A to the tree be n. Then find the distance from B to the tree in terms of the distance of AB and n. Express the height of the tree, h, using two different trigonometric ratios; then equate them.)



Check your answers by turning to the Appendix, page 117.

Sine and Cosine Ratios

Look again at the triangles you drew on the grid at the start of this activity.

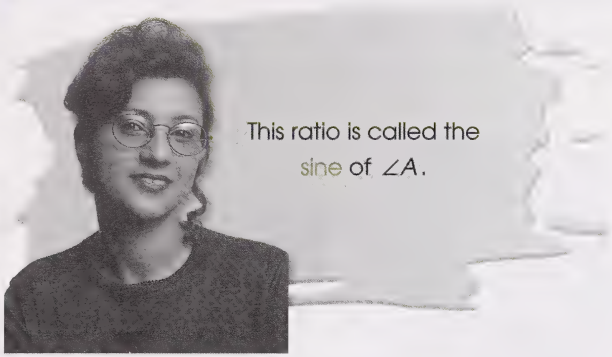


You are going to investigate two additional ratios with reference to $\angle A$.

21. **a.** Using $\angle A$ as the reference angle, find the ratio of the opposite side, BC to the hypotenuse, AB. Do the same for ΔADE and ΔAFG. Use your calculator to find the decimal value for the ratio for each triangle.

b. Are the ratios the same for each triangle?

Check your answers by turning to the Appendix, page 117.



If $\angle A$ is an acute angle in a right triangle, $\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$. Often, you will see the sine ratio of a given angle, A, written in short form as $\sin A = \frac{\text{opp}}{\text{hyp}}$.

Since $\angle A$ is the same angle you used when you found the tangent ratio, its measure must be approximately 37°. Use your calculator to find angle A from the sine ratio. Enter the following keystrokes on your calculator.

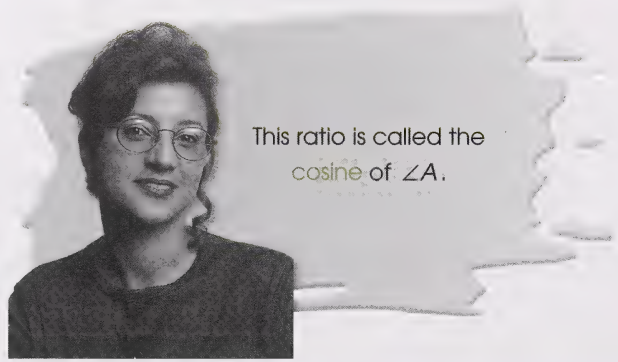
2nd | sin | (| 3 | ÷ | 5 |) | =

22. What value appears in the display? Round the value to the nearest degree. What angle is this value?

23. a. Using $\angle A$ as the reference angle, find the ratio of the adjacent side, AC to the hypotenuse, AB. Do the same for ΔADE and ΔAFG. Use your calculator to find the decimal value for the ratio for each triangle.

b. Are the ratios the same for each triangle?

Check your answers by turning to the Appendix, page 117.



If $\angle A$ is an acute angle in a right triangle, $\cos A = \frac{\text{length of side adjacent } \angle A}{\text{length of hypotenuse}}$. Often, you will see the cosine ratio of a given angle, A, written in short form as $\cos A = \frac{\text{adj}}{\text{hyp}}$.

24. Use a scientific calculator to find $\angle A$ from the cosine ratio. Write out the required keystrokes and the resulting display. Round the result to the nearest degree.

25. Is the measure of $\angle A$ the same as those you obtained using the tangent and sine ratios?

Check your answers by turning to the Appendix, page 118.

You can use a scientific calculator to find the sine and cosine ratios for a given angle in the same way you found the tangent ratio of a given angle earlier in this activity.

Example

Find the sine and cosine of 49° using a scientific calculator. Round your answers to 4 decimal places.

Solution

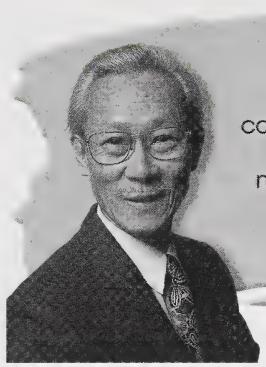
sin 4 9 =

0.75470958

cos 4 9 =

0.656059029

$\therefore$ $\sin 49° = 0.7547$ and $\cos 49° = 0.6561$



Remember: Your calculator may require you to enter the number first. Consult your manual.

Use a scientific calculator to answer questions 26 and 27.

26. Find the sine and cosine ratios for the following angles. Round the values to 4 decimal places where necessary.

a. 30° **b.** 87° **c.** 9° **d.** 45° **e.** 60°

27. Study the sine and cosine ratios for the angles in question 26.

a. Predict the value of $\sin 0°$, $\sin 90°$, $\cos 0°$, and $\cos 90°$. Check your predictions with your calculator.

b. For what angle are the sine and cosine ratios the same?

Check your answers by turning to the Appendix, page 118.

You can use a scientific calculator to find an angle given the sine or cosine ratio. Look at the following examples.

Example

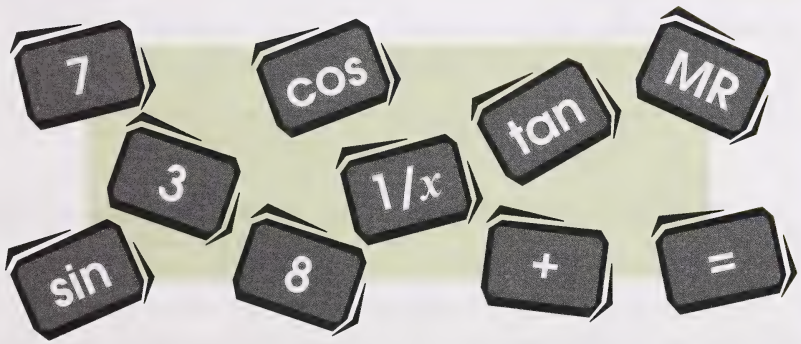
If $\sin A = 0.3845$, find the measure of $\angle A$. Round your answer to the nearest degree.

Solution

2nd sin • 3 8 4 5 =

22.6 12703 16

$\therefore \angle A \doteq 23°$



Example

If $\cos A = 0.1959$, find the measure of $\angle A$. Round your answer to the nearest degree.

Solution

2nd cos • 1 9 5 9 =

78.702696 12

$\therefore \angle A \doteq 79°$

Example

If $\sin A = \frac{11}{17}$, find the measure of $\angle A$. Round your answer to the nearest degree.

Solution

2nd sin (1 1 ÷ 1 7) =

40.3202 1507

$\therefore \angle A \doteq 40°$

Use a scientific calculator to answer question 28.

28. Find the angle that is equivalent to the following trigonometric ratios. Round your answers to the nearest degree.

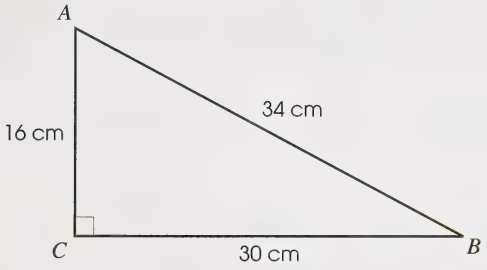
a. $\sin C = 0.82$

b. $\cos E = 0.0814$

c. $\sin A = \frac{5}{8}$

d. $\cos B = \frac{16}{23}$

29. Use the diagram given to answer the questions that follow.



a. Find the sine and cosine ratios for each acute angle. Round your answers to 4 decimal places.

b. What do you notice about the ratio for $\sin A$ and the ratio for $\cos B$? What about $\sin B$ and $\cos A$?

Check your answers by turning to the Appendix, page 119.

You solved problems in Section 1 concerning right triangles by using the Pythagorean Theorem. The type of problem you could solve was limited. Now, using trigonometric ratios, you can solve a number of additional types of problems involving right triangles.

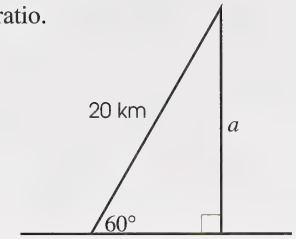
Example

A rocket leaves the launching pad and moves along a straight trajectory inclined at an angle of 60° to the ground. If it travels along this line for the first 20 km, what is its altitude at that point?

Solution

Draw a diagram.

Find the altitude, a, using a trigonometric ratio.

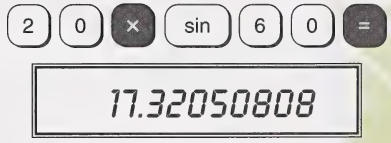


$$\sin 60° = \frac{a}{20}$$

$$20(\sin 60°) = 20\left(\frac{a}{20}\right)$$

$$20 \sin 60° = a$$

$$a = 20 \sin 60°$$



The rocket is about 17.3 km above the ground after travelling 20 km through the air.

Now solve the following problems.

30. Given the following data about the Jasper Tramway, find the approximate angle of inclination (elevation) of the lift. Round your answer to the nearest degree.

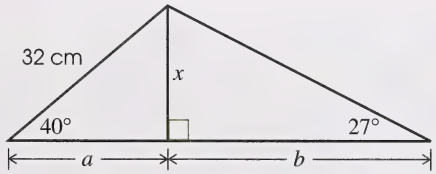
- Lower terminal: 1408 m above sea level
- Upper terminal: 2500 m above sea level
- Length of track: 2000 m

31. An airplane flying at a speed of 50 m/s increases its altitude at a rate of 7.5 m/s. Find the angle at which the airplane is climbing. Round your answer to the nearest degree.

32. How far up the wall will an 8-m ladder reach if it must be inclined at 65° with the ground for safety. Round your answer to 2 decimal places.

33. The altitude of an equilateral triangle is 19 cm. Find the length of each side of the triangle. Round your answer to the nearest centimetre.

34. Find the measure of a and b to the nearest centimetre.



35. Karl was flying his kite on Saturday morning. He wanted to know how high his kite was flying above the ground. He attached the kite's 45-m string to the ground and then paced off 27 m to a point where the kite was flying directly over his head.

a. What angle did the kite's string form with the ground? Round your answer to the nearest degree.

b. What angle did the kite's string form with a vertical line from the kite to the ground?

c. How high was Karl's kite from the ground? Round your answer to the nearest metre.

d. Describe other methods you could have used to answer question 35.c.

Check your answers by turning to the Appendix, page 120.

Did You Know?

The word *trigonometry* is derived from three Greek words.

tri	*gono*	*metry*
three	angle	measurement

Trigonometry has important applications in surveying, navigation, construction work, electronics, advanced chemistry, and other occupational fields.

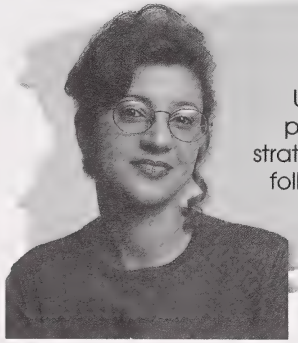
To help you recall the three primary trigonometric ratios, you might use an acronym like the following:

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
SOH	*CAH*	*TOA*
Soak	A	Toe

This phrase may help you remember the letter arrangements, or you may be able to come up with one you prefer.

Use whatever helps you to remember these ratios.

Now Try This

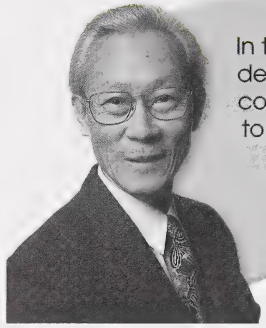


Use one of the problem-solving strategies to solve the following problem.

36. Shawna walked 60 m east, then 20 m north, then 20 m east, and then 40 m north. How far is she from her starting point (as measured in a straight line)?

Check your answers by turning to the Appendix, page 124.

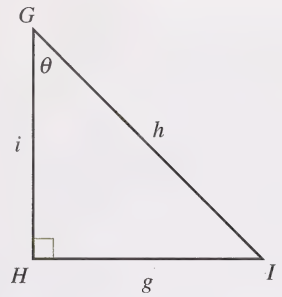
Looking Back



In this activity, you worked with the definition of the tangent, sine, and cosine ratios. You used these ratios to find a missing side or angle in a right triangle and to solve practical problems.

37. In your journal, write the acronym SOH CAH TOA. What does this acronym represent?

38. Copy the following triangle into your journal.



Write the ratios for $\sin\theta$, $\cos\theta$, and $\tan\theta$ in terms of h, i and g.

39. If $\sin\theta = 0.77$, write the keystrokes used on a calculator to determine θ.

Check your answers by turning to the Appendix, page 124.

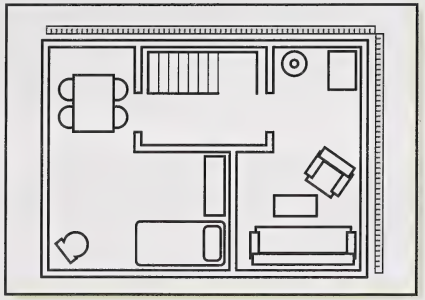
Activity 5: Enlargements and Reductions

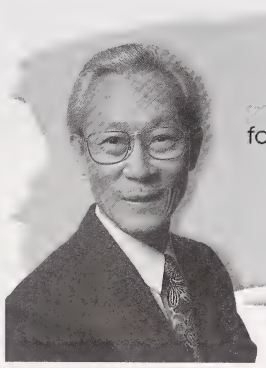
Have you ever had a photograph enlarged? In the following photographs, notice that only size is altered—the shape remains unchanged. In fact, for all enlargements and reductions, the size may change, but the shape remains the same.

In mathematics, when a figure is enlarged or reduced, the transformation is called a dilatation.

In Activity 3, you explored similar triangles. For similar triangles, one triangle can be viewed as an enlargement or reduction of the other. Your work with similar triangles can be extended to dilatations.

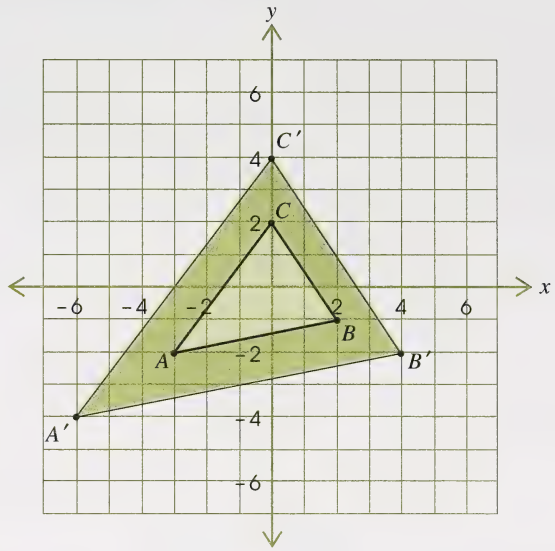
Some people's jobs depend on dilatations. For example, an architect—a person who creates or works with blueprints, floorplans, and model houses and buildings—makes a living working with dilatations.





Complete the following questions to help you explore dilatations.

1. ΔABC —with coordinates $A(-3,-2)$, $B(2,-1)$, and $C(0,2)$—and $\Delta A'B'C'$ —with coordinates $A'(-6,-4)$, $B'(4,-2)$, and $C'(0,4)$—are given on the following coordinate grid.



a. Measure and compare the lengths of the corresponding sides of ΔABC and $\Delta A'B'C'$.

b. Measure and compare the corresponding angles of ΔABC and $\Delta A'B'C'$.

c. What else can you say about the corresponding sides of the two triangles?

Check your answers by turning to the Appendix, page 124.

Properties of a Dilatation

- The image is similar to the original figure.
- The orientation of the image and the original figure are the same.
- The corresponding sides of the image and the original figure are parallel.

The scale factor of a dilatation is the ratio of the lengths of the sides of the image to the lengths of the sides of the original figure. You can use a mapping rule to give a scale factor.

$(x, y) \rightarrow (kx, ky)$, where k is any number

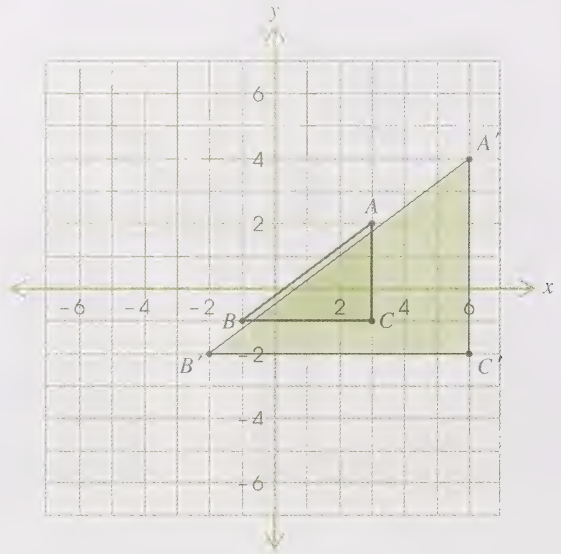
This mapping rule states that the new figure will have coordinates (kx, ky).

2. What is the scale factor for the diagram in question 1?

3. Write the mapping rule for the dilatation in question 1.

4. Figure *WXYZ* has vertices $W(-1, 2)$, $X(3, 2)$, $Y(3, -1)$, and $Z(-2, -1)$. Draw the figure on a coordinate grid. Using a scale factor of 2, write the coordinates of and draw the image of *WXYZ*.

5. A dilatation $(x, y) \rightarrow (2x, 2y)$ is applied to ΔABC to get $\Delta A'B'C'$.



a. Find the areas of ΔABC and $\Delta A'B'C'$.

b. Write a statement about the relationship between the two areas in question 7.a.

6. **a.** Draw ΔDEF with vertices $D(0, 4)$, $E(-2, 0)$, and $F(2, 0)$ on a coordinate grid; then find its image using a scale factor of $\frac{1}{2}$.

b. Write the mapping rule for the dilatation in question 5.a.

7. Figure *ABCD* has vertices $A(-1, 1)$, $B(-1, -2)$, $C(2, -2)$, and $D(2, 1)$. Draw *ABCD* on a coordinate grid and find its image under the mapping rule $(x, y) \rightarrow (3x, 3y)$.

8. **a.** When a dilatation is applied to a figure, what changes?

b. When a dilatation is applied to a figure, what remains the same?

9. Copy and complete the following chart in your notebook. Fill in the blanks for dilatations of points about $(0, 0)$.

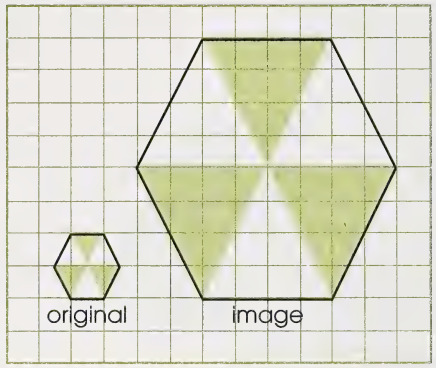
Original	Image	Mapping Rule
$(3, -1)$		$(x, y) \rightarrow (2x, 2y)$
$(4, 2)$	$(2, 1)$	
	$(-2, 3)$	$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$
$(4, 6)$	$(6, \quad)$	
$(\quad, 2)$	$(9, \quad)$	$(x, y) \rightarrow (3x, 3y)$

10. Copy and complete the following chart in your notebook.

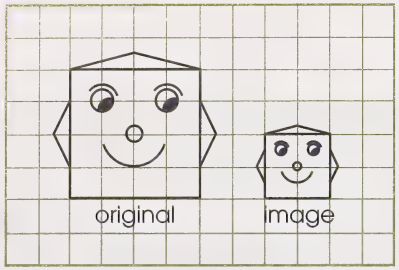
Area of Original	Area of Image	Scale Factor
24 cm^2	6 cm^2	
16 cm^2	36 cm^2	
	4 cm^2	$\frac{1}{3}$
8 cm^2		2.5

11. A figure and its image are shown in each of the following pairs. Determine the scale factor.

a.

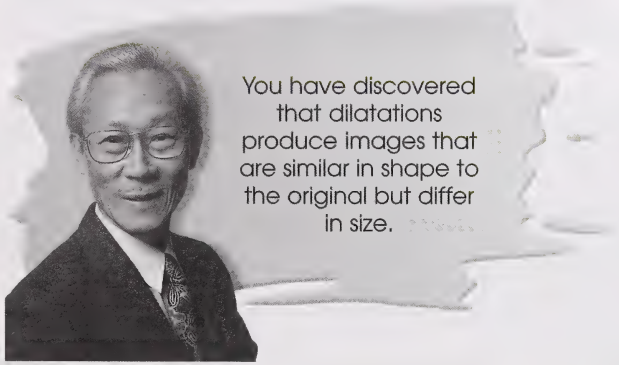


b.



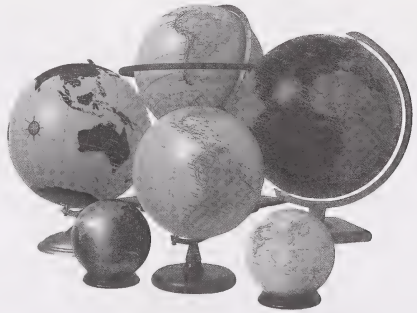
Check your answers by turning to the Appendix, page 125.

Now Try This



You have discovered that dilatations produce images that are similar in shape to the original but differ in size.

12. Globes are examples of a real-world dilatation. A globe is a reduction of Earth. Give at least three real-world examples of dilatations. State whether each is an enlargement or a reduction.



Check your answers by turning to the Appendix, page 126.

Looking Back

In this activity, you enlarged and reduced a shape. You noticed the shape remained the same while the size changed.

13. If the mapping rule is $(x, y) \rightarrow (kx, ky)$, where $0 < k < 1$, would the new shape be enlarged or reduced compared to the original? Explain.

Check your answers by turning to the Appendix, page 126.

Conclusion

In this section, you used the concepts involved in congruent and similar triangles to develop the primary trigonometric ratios. In turn, you used the primary trigonometric ratios to solve problems involving right triangles. You also constructed tools to help you apply your knowledge of trigonometry to solve practical problems involving measurement and right triangles.

Finally, you used your knowledge of similar triangles to explore dilatations.

After completing this section, you can probably use the concepts you studied to find the height of any tower or tall building. Would you be able to find the height of the building in the photograph using a clinometer?

Assignment

Turn to Assignment Booklet 5B and complete the assignment for Section 2.

Module Summary

In Module 5, you worked with Pythagoras's triangle and solved a variety of problems involving the right triangle. Your work with the Pythagorean Theorem provided a basis for your study of trigonometry.

In trigonometry, you defined the primary trigonometric ratios and used them to solve right triangles. You also explored how to make indirect measurements using trigonometric ratios and simple, homemade measuring instruments. You then enlarged and reduced figures using the concept of similar shapes.

The triangle and trigonometry are essential parts of many professionals' mathematical toolkits. The architect, surveyor, and engineer are just a few of the professionals that use mathematics daily to create today's modern, technological world.

As you progress through your study of mathematics, you will repeatedly encounter the ideas to which you were introduced in this module.

Final Module Assignment

Turn to Assignment Booklet 5B and complete the final module assignment for Module 5.

COURSE SURVEY FOR MATHEMATICS PREPARATION 10
(© 2000)

After you have completed the assignments in this course, please fill out this questionnaire and mail it to the address given on the last page. This course is designed in a new distance learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated, as future course revisions can then incorporate any necessary improvements.

Name _____ Age ❑ under 19 ❑ 19 to 40 ❑ over 40

Address _____ File No. _____

_____ Date _____

_____

Design

1. This course contains a series of Student Module Booklets. Do you like the idea of separate booklets?

   _____

   _____

2. Have you ever enrolled in a correspondence or distance learning course that arrived as one large volume?

   ❑ Yes ❑ No If yes, which style do you prefer?

   _____

   _____

3. The Student Module Booklets contain a variety of self-assessed activities. Did you find it helpful to be able to check your work and have immediate feedback?

   ❑ Yes ❑ No If yes, explain.

   _____

   _____

4. Were the questions and directions easy to understand?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

5. Each section contains follow-up activities. Which type of follow-up activity did you choose?

   ❑ mainly extra help
   ❑ mainly enrichment
   ❑ a variety
   ❑ none

   Did you find these activities beneficial?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

6. Did you understand what was expected in the Assignment Booklets?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

7. The course materials were designed to be completed by students working independently at a distance. Were you always aware of what you had to do?

   ❑ Yes ❑ No If no, provide details.

   _____

   _____

8. This distance learning course may include an assortment of drawings, photographs, and charts.

   a. Did you find the visuals in this course helpful?

      ❑ Yes ❑ No Comment on the lines below.

      _____

      _____

   b. Did you find the variety of visuals in this course motivating?

      ❑ Yes ❑ No Comment on the lines below.

      _____

      _____

9. Suggestions for audiocassette, videocassette, and computer activities may have been included in the course. Did you complete these media activities?

   ❑ Yes ❑ No Comment on the lines below.

   _____

   _____

Only students enrolled in a junior high course need to complete the following question.

10. The Student Module Booklet may have directed you to work with your teacher. How well did you work as a team?

    Student's comments: _____

    _____

    _____

    _____

    Teacher's comments: _____

    _____

    _____

    _____

Course Content

1. Was enough detailed information provided to help you learn the expected skills and ojectives?

   ❑ Yes ❑ No Comment on the lines below.

   _____

   _____

2. Did you find the workload reasonable?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

3. Did you have any difficulty with the reading level?

❑ Yes ❑ No Please comment.

_____

_____

4. How would you assess your general reading level?

❑ poor reader ❑ average reader ❑ good reader

5. Was the material presented clearly and with sufficient depth?

❑ Yes ❑ No If no, explain.

_____

_____

General

1. What did you like least about the course?

_____

_____

2. What did you like most about the course?

_____

_____

Additional Comments

_____

_____

_____

_____

_____

_____

_____

Only students enrolled with the Alberta Distance Learning Centre need to complete the remaining questions.

1. Did you contact the Alberta Distance Learning Centre for help or information while doing your course?

   ❑ Yes ❑ No If yes, approximately how many times? _____

   Did you find the staff helpful?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

2. Were you able to fax any of your assignment response pages?

   ❑ Yes ❑ No If yes, comment on the value of being able to do this.

   _____

   _____

3. If you mailed your assignment response pages, how long did it take for their return?

   _____

   _____

4. Was the feedback you received from your correspondence or distance learning teacher helpful?

   ❑ Yes ❑ No Please comment.

   _____

   _____

   _____

   _____

Thanks for taking the time to complete this questionnaire. Your feedback is important to us. Please return this questionnaire to the address on the right.

If you are enrolled at the Alberta Distance Learning Centre and have been mailing your Assignment Booklets to ADLC, you may return this questionnaire with the final Assignment Booklet in the course.

Instructional Design and Development
Learning Technologies Branch
Box 4000
Barrhead, Alberta
T7N 1P4

Appendix

Problem-Solving Strategies
Glossary
Suggested Answers
Credits
Cut-out Learning Aids

Problem-Solving Strategies

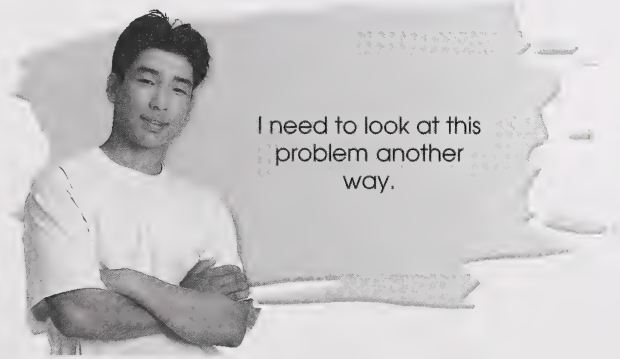
Be sure to study the following problem-solving strategies and use them when you encounter problems in this course.

Whichever method of problem solving you may choose to use, consider the following:

- Show that you understand the problem by showing all the steps needed for finding the answer.
- Draw and neatly label any diagram, graph, or chart that may help you answer the problem.
- Write a concluding sentence that answers the question being asked.

Changing Your Point of View

Sometimes you find that you cannot solve a particular problem because of your "mind set." Perhaps you made an assumption about the problem that is incorrect. If your attempts to solve a problem are not successful, it is often helpful to try to change your point of view.



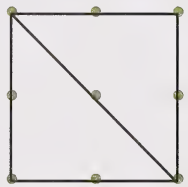
Example

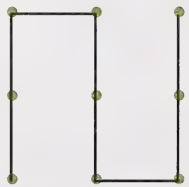
Draw four line segments through the following nine dots without lifting your pencil from the paper and without retracing your path.

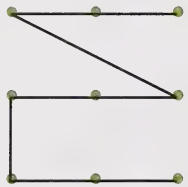
Solution

Step 1: You make several attempts at solving the problem.

5 line segments

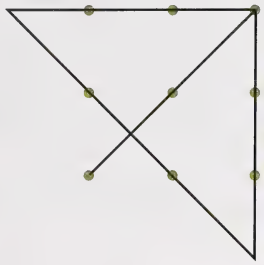

5 line segments


5 line segments


5 line segments

Step 2: You realize you are blocked in your attempts. You have made the assumption that the dots must be connected in a way similar to a child's dot-to-dot picture.

In order to solve the problem, you must change your point of view and realize that the line segments may lie outside the confines of the dots.



4 line segments

Using Objects

Objects can be used to help you solve a problem.

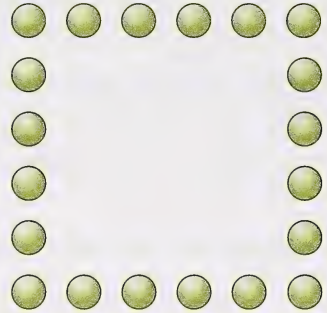
Example

To enclose a square lot with a fence, 20 posts are used. If the posts are placed 3 m apart, what is the length of one side of the lot?

Solution

Step 1: Use objects such as blocks, checkers, or coins to represent the posts.

The advantage of using objects to represent the posts is that you can easily rearrange them.



Step 2: Count the number of spaces between the posts on one side of the square.

There are five spaces between the six posts on one side of the square.

Step 3: Calculate the length of one side of the square.

$$5 \times 3 = 15$$

The length of one side of the lot is 15 m.

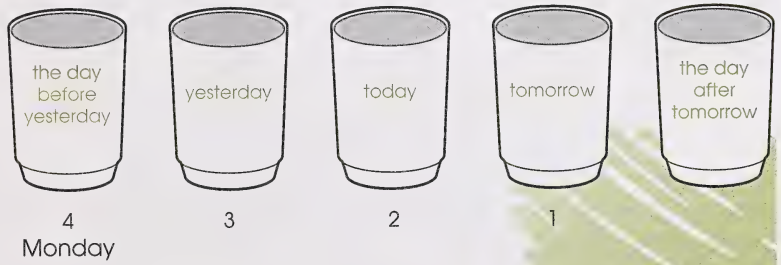
Example

Four days before the day after tomorrow was Monday. What day of the week is it today?

Solution

Step 1: Use paper cups to represent the days. The cups could be labelled as shown.

Step 2: Begin at the cup representing the day after tomorrow and count back four days. That day was Monday.



If Monday was the day before yesterday, yesterday was Tuesday, and today is Wednesday.

Using Diagrams

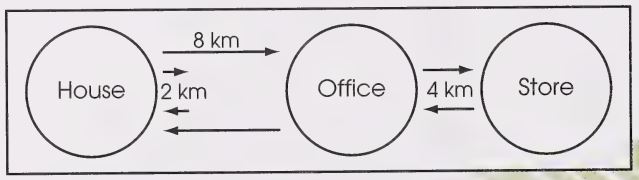
You can use sketches to solve problems.

Example

Chris's office is 8 km from his home. Yesterday morning he drove 2 km before he realized that he had forgotten his briefcase. He returned home to get his briefcase and then drove to his office. At noon, Chris drove 4 km to a store. He then went back to the office for the rest of the afternoon. At the end of the day, he drove straight home. How far did he drive yesterday?

Solution

Step 1: Draw and label a diagram to help you understand the problem.



Step 2: Calculate the distance Chris drove.

$$2+2+8+4+4+8=28$$

Chris drove 28 km.

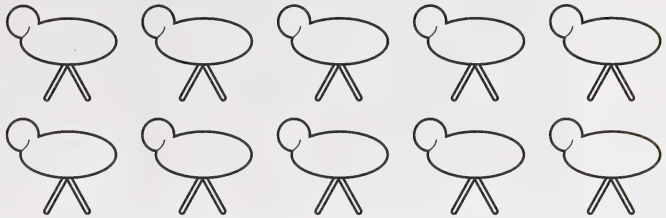
Example

There are 10 animals in a barnyard. Some are chickens and the rest are sheep. Ruth counts 28 legs. How many chickens and how many sheep are there? **Hint:** Chickens have 2 legs and sheep have 4 legs.

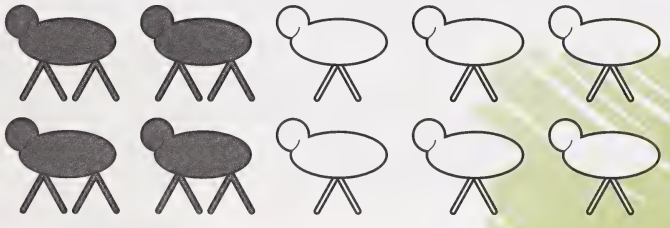
Solution

Draw a diagram to help you solve the problem.

Step 1: Every animal has at least 2 legs. So, draw a diagram of 10 animals, each of which has at least 2 legs.



Step 2: The group of animals in the problem had 28 legs. So, you need to add 8 legs to the drawing. Add the legs in pairs to the sketch of each animal.



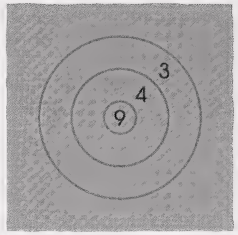
There are 4 sheep and 6 chickens.

Making an Organized List

Some problems require you to list all the possible solutions. It is important to make an organized list so that you do not miss any possibilities.

Example

Julio and his friends like playing a game called Bull's Eye with lawn darts. They draw a target in the dirt and assign values to the three regions of the target.



If Julio throws three darts and all three hit the target, how many different point totals are possible?

Solution

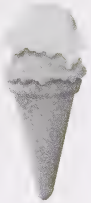
Begin with the highest possible score and list all the possibilities in order.

$9+9+9=27$ ← highest possible score
$9+9+4=22$
$9+9+3=21$
$9+4+4=17$
$9+4+3=16$
$9+3+3=15$
$4+4+4=12$
$4+4+3=11$
$4+3+3=10$
$3+3+3=9$ ← lowest possible score

There are 10 possible point totals.

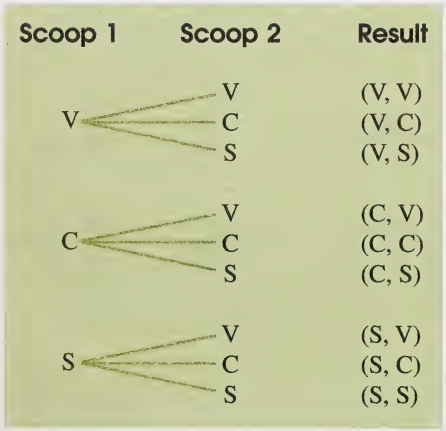
Example

At an ice-cream parlour there are 3 flavours of ice cream: chocolate, vanilla, and strawberry. How many different double-scoop ice-cream cones are possible? **Hint:** A chocolate-vanilla cone is different from a vanilla-chocolate one. Order makes a difference.



Solution

Draw a tree diagram to help you make an organized list. Use V for vanilla, C for chocolate, and S for strawberry.



Nine different double-scoop ice-cream cones are possible.

Using Venn Diagrams

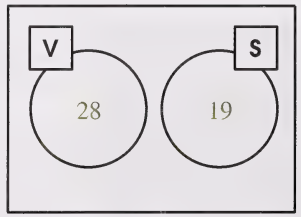
A type of diagram that is helpful in some problems is a Venn diagram. In this special kind of diagram, circles are used to represent groups of people, animals, or objects that have certain characteristics. The positioning of the circles in relation to one another represents relationships among these groups. These diagrams were named after an English mathematician by the name of John Venn (1834–1923).

Example

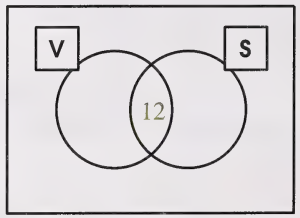
There are 28 girls on the volleyball team and 19 girls on the swim team. If 12 girls belong to both teams, how many girls belong only to the swim team?

Solution

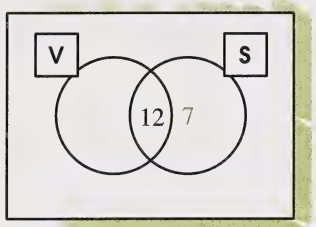
Step 1: Represent the members of the volleyball team with a circle. Represent the members of the swim team with another circle.



Step 2: Represent the members of both teams by the intersection of the two circles.



Step 3: Now, calculate the number of girls who belong only to the swim team.



$19 - 12 = 7$

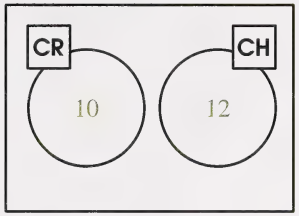
So, 7 girls belong only to the swim team.

Example

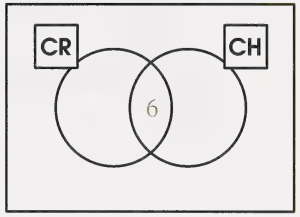
Reilly bought some plates at a garage sale. He discovered that 10 plates were cracked, 12 plates were chipped, 6 plates were both chipped and cracked, and 2 plates were neither chipped nor cracked. How many plates did Reilly buy?

Solution

Step 1: The problem states that 10 plates were cracked (CR) and 12 plates were chipped (CH). Represent the cracked plates with a circle. Represent the chipped plates with a second circle.

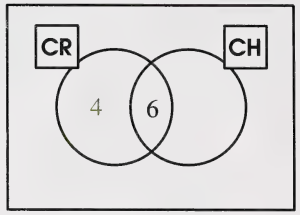


Step 2: The problem states that 6 plates were both chipped and cracked. Represent the cracked and chipped plates with the intersection of the two circles.



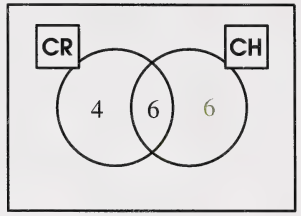
Step 3: Calculate the number of plates that were only cracked.

$10 - 6 = 4$

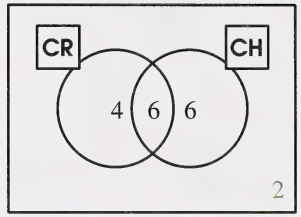


Step 4: Calculate the number of plates that were only chipped.

$12 - 6 = 6$



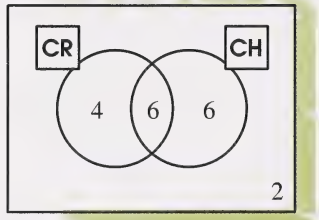
Step 5: The problem states that 2 plates were neither chipped nor cracked. Represent the undamaged plates with the region outside the circles.



Step 6: Calculate the total number of plates that were bought.

$4 + 6 + 6 + 2 = 18$

So, 18 plates were bought.



Example

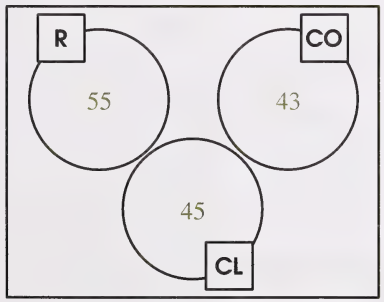
In a survey of 100 people, the following information was collected:

- 43 people like country music
- 55 like rock music
- 45 like classical music
- 8 people like all three forms
- 15 people like both country and classical music
- 25 like both rock and country music
- 20 like both rock and classical music

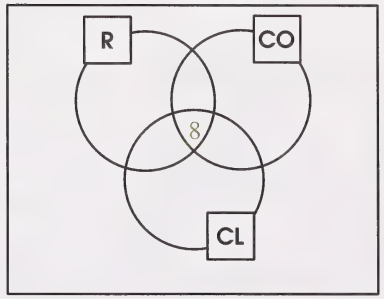
How many people do not like any of these forms of music?

Solution

Step 1: The problem states that 55 people like rock music (R), 43 like country music (CO), and 45 like classical music (CL). Use three circles to represent these preferences.

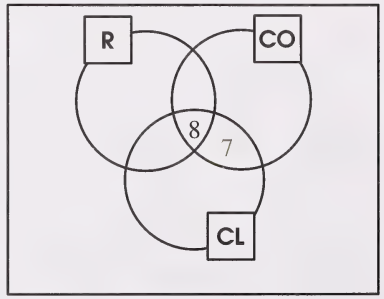


Step 2: The problem states that 8 people like all three forms. The intersection of the three circles shows the people who like all three forms.



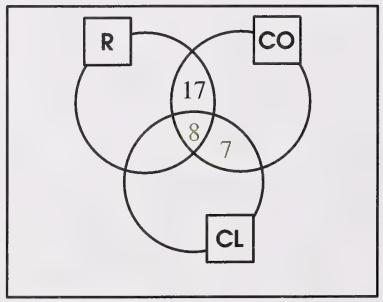
Step 3: The problem states that 15 people like both country and classical music. Calculate the number of people who like country and classical music, but not rock music.

$$15 - 8 = 7$$



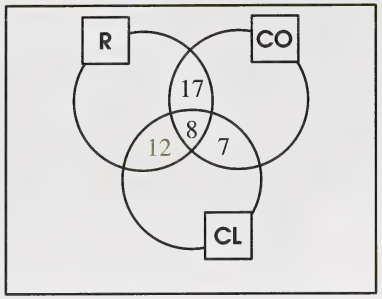
Step 4: The problem states that 25 people like both rock and country music. Calculate the number of people who like both country and rock music, but not classical music.

$$25 - 8 = 17$$



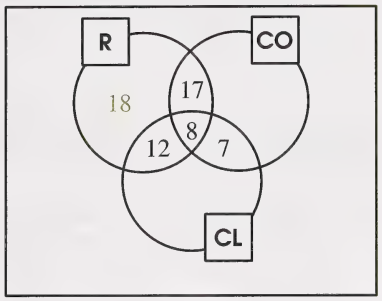
Step 5: The problem states that 20 people like both rock and classical music. Calculate the number of people who like both rock and classical music, but not country music.

$$20 - 8 = 12$$



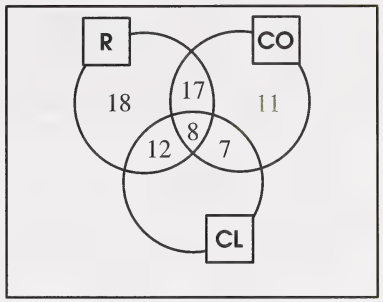
Step 6: Now that you know the number of people who like rock music and any other form of music, calculate the number of people who only like rock.

$$55 - (17 + 8 + 12) = 18$$



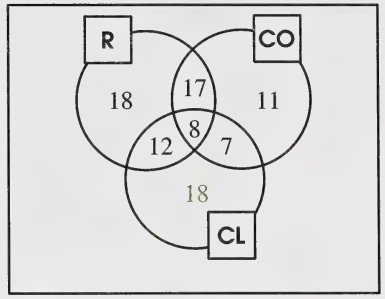
Step 7: Now that you know the number of people who like country music and any other form of music, calculate the number of people who only like country music.

$$43 - (17 + 8 + 7) = 11$$



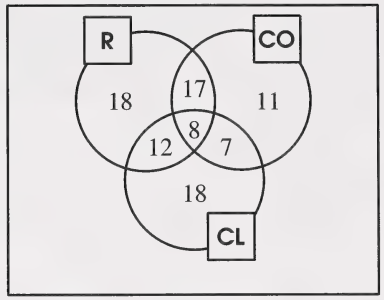
Step 8: Now that you know the number of people who like classical music and any other form of music, calculate the number of people who only like classical music.

$$45 - (12 + 8 + 7) = 18$$



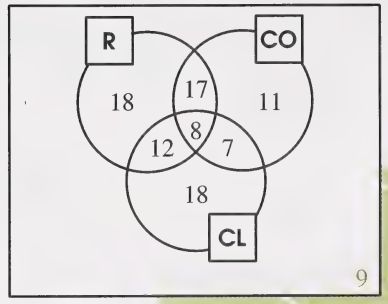
Step 9: Now, you can calculate the total number of people who like rock, country, or classical music.

$$18 + 17 + 8 + 12$$
$$+ 11 + 7 + 18 = 91$$



Step 10: The problem states that 100 people were surveyed. So, you can calculate the number of people who do not like any of these forms of music.

$$100 - 91 = 9$$



Of the 100 people surveyed, 9 people do not like any of these forms of music.

Making a Table

You can use a table to organize information.

Example

Hannah works at her mother's store after school. She earns 20¢ for each customer that she assists and 35¢ for each bag that she packs. One day she did 19 jobs and earned $4.85. How many bags did she pack and how many customers did she assist that day?

Solution

Write down all possible combinations of 19 jobs until you find a combination that totals $4.85. Use a table to organize the information.

In the first column, put 1 bag packed and 18 customers assisted. Calculate the money received for that.

$$\begin{aligned} 1 \times 35¢ &= \$\ .35 \\ 18 \times 20¢ &= \$3.60 \\ \text{Total} \quad &\$3.95 \end{aligned}$$

Then complete the table for other combinations.

Packing	1	2	3	4	5	6	7
Assisting	18	17	16	15	14	13	12
Earned	$3.95	$4.10	$4.25	$4.40	$4.55	$4.70	$4.85

correct guess

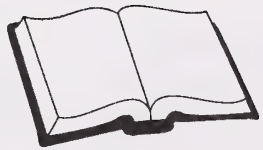
Hannah packed 7 bags and assisted 12 customers that day.

Guessing, Checking, and Revising

Many mathematical problems can be solved by systematic trial or guessing, checking, and revising.

Example

Shelley opened her math textbook and noticed that the product of the numbers of the two open pages was 2970. To what pages was the book opened? **Hint:** The left-hand page number of a book is even; the right-hand page number is odd.



Solution

Method 1: Using Paper and Pencil

Step 1: Note the conditions that need to be met.

- The left-hand page number must be even.
- The right-hand page number is one more than the left-hand page number.
- The product of the numbers must be 2970.

Step 2: Make a guess; check your guess; and revise if necessary.

It is helpful to organize your guesses in a table.

Left-Hand Page Number	50	52	54
Right-Hand Page Number	51	53	55
Product of Page Numbers	2550	2756	2970

correct guess

So, the book was opened to pages 54 and 55.

Method 2: Using a Computer

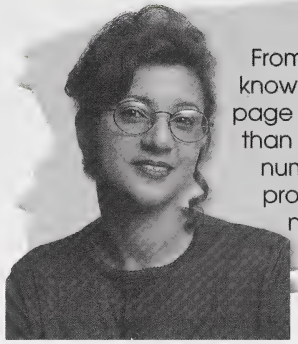
Use a computer and a spreadsheet program to help you solve the problem.

Step 1: Label the cells in the spreadsheet. **Hint:** The cell address at the top of the spreadsheet shows which cell is active. There is also a border around the active cell. You don't type directly in this cell; instead, you use the entry bar at the top of the spreadsheet.

A5 ✗ ✓ Product of page numbers

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	
4	Right-hand page number	
5	Product of page numbers	
6		

Step 2: Enter either a number or a formula for each of the cells.



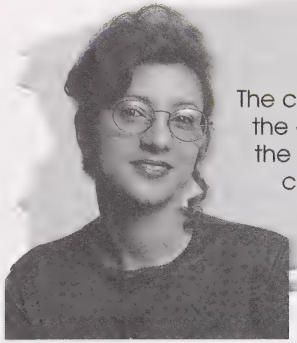
From the problem, you know that the right-hand page number is one more than the left-hand page number and that the product of the page numbers is 2970.

Here is what you enter:

- Begin with cell B3 and enter a guess of 50.
- Next, enter "=B3+1" in cell B4. This means the number in the B4 cell will be one more than the number in the B3 cell.
- Finally, enter "=B3*B4" in cell B5. This means that the number in the B5 cell will be the product of the numbers in the B3 and B4 cells.

B5 ✗ ✓ =B3*B4

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	50
4	Right-hand page number	51
5	Product of page numbers	2550
6		



The computer performs the calculations and the results appear in cells B4 and B5.

Step 3: Check your initial guess, and revise it if necessary. The product is 2970, not 2550; therefore, revise your guess.

Delete the number in the B3 cell and enter a guess of 54.

B3	✗ ✓	54

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	54
4	Right-hand page number	55
5	Product of page numbers	2970
6		

The computer performs the calculations and different results appear in cells B4 and B5.

Step 4: Check your initial guess, and revise it if necessary. The product is 2970.

So, the book was opened to pages 54 and 55.

Acting Out a Problem

For some problems, you may find it helpful to physically act out the problem situation.

Example

How many cards does each boy have?

Solution

Step 1: Ask yourself these questions:

- What would happen if Jon gave Matt a card? The first condition to be met is that they must have an equal number of cards after the exchange.

- What would happen if Matt gave Jon a card? The second condition to be met is that Jon will have twice as many cards as Matt after the exchange.

Step 2: Make a guess and act out the problem. If your guess does not fit the conditions, revise your guess and repeat the process.

It is helpful to organize your guesses in a table.

Matt's cards	1	2	3	4	5
Jon's cards	3	4	5	6	7

correct guess

So, Matt has 5 cards and Jon has 7 cards.

Note

If Jon gives Matt a card, each boy will have an equal number of cards after the exchange, 6.

If Matt gives Jon a card, Jon will have twice as many cards as Matt after the exchange. (Jon will have 8 cards and Matt will have 4 cards.)

Working Backwards

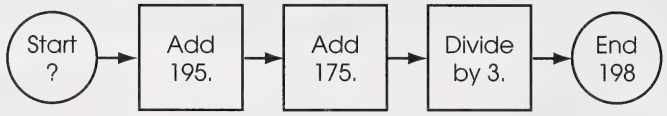
In mathematics, you are usually given information and then asked to find the answer. Sometimes, however, you are given the answer and then asked to find a piece of information. In cases like these, you need to work backwards.

Example

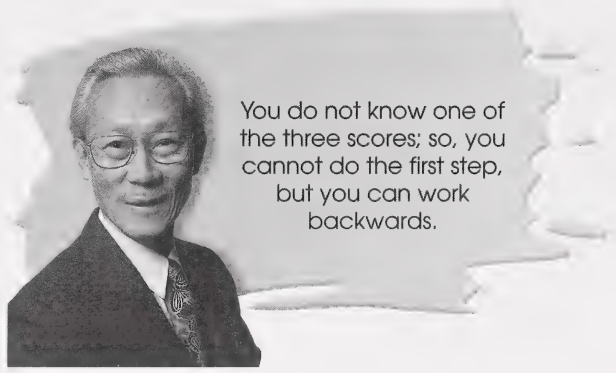
Ross plays in a bowling league. His average in the league is 198. Today Ross scored 195 and 175 in his first two games. What must he score in his third game to maintain his average?

Solution

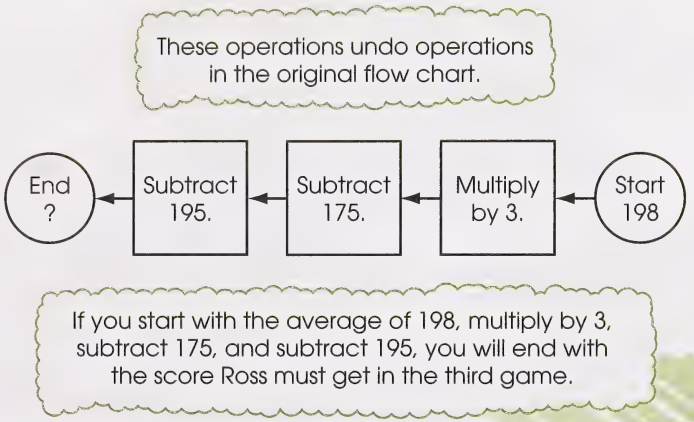
Step 1: Consider how the problem would be done if you worked forwards. This flow chart shows the sequence of the steps.



You start with the score of the third game, add each of the scores of the other two games, divide by 3, and end with the average of 198.



Step 2: Use a reverse flow chart to work backwards.

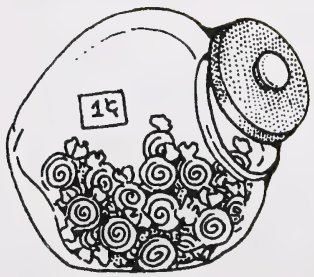

These operations undo operations in the original flow chart.
End ?
Subtract 195.
Subtract 175.
Multiply by 3.
Start 198
If you start with the average of 198, multiply by 3, subtract 175, and subtract 195, you will end with the score Ross must get in the third game.

Ross must score 224 in the last game to maintain his average.

Note: You can check to see if 224 is the correct third score by using the flow chart in Step 1.

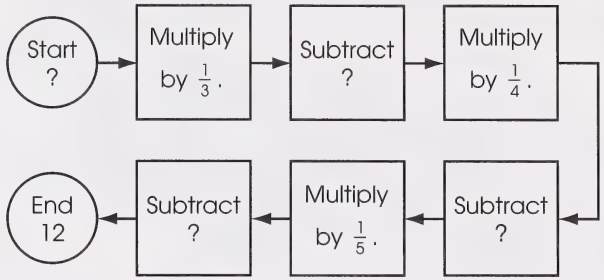
Example

Ruth has a jar of candies. She gives Raschid one-third of her candies. She then gives Steven one-fourth of the remaining candies. Finally, she gives one-fifth of the candies that she has left to Rachel. If Ruth has 12 candies left at the end, how many did she have at the beginning?



Solution

Step 1: Use a flow chart to show the order of operations.

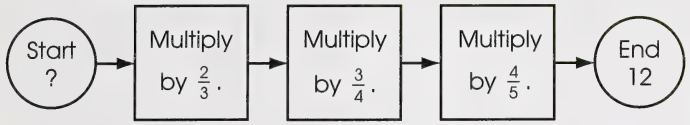


This flow chart is **not** helpful because you do not know how much to subtract in the second, fourth, and sixth boxes.

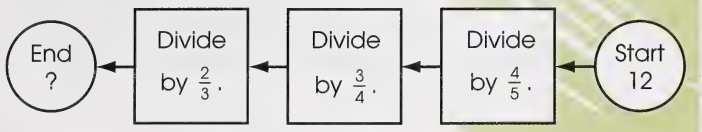
Knowing the following relationships will help you draw a more complete flow chart.

- Giving away one-third of the candies is the same as multiplying the number of candies by two-thirds. $1-\frac{1}{3}=\frac{2}{3}$
- Giving away one-fourth of the remaining candies is the same as multiplying by three-fourths. $1-\frac{1}{4}=\frac{3}{4}$
- Giving away one-fifth of the remaining candies is the same as multiplying by four-fifths. $1-\frac{1}{5}=\frac{4}{5}$

The following flow chart is more useful.



Step 2: Use a reverse flow chart to work backwards.



Ruth had 30 candies at the beginning.

Simplifying a Problem

Are you sometimes confused about which operations to perform in a problem because the numbers in the problem are very large or because the problem seems very complicated? There are strategies you can use to simplify a problem.

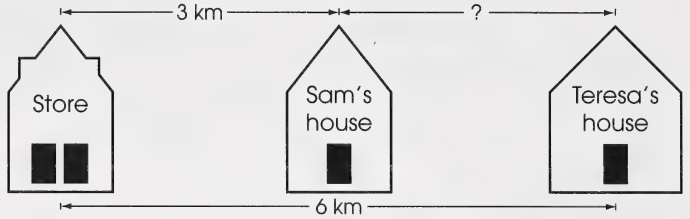
Example

The average distance from the Sun to Mars is 228 000 000 km, and the average distance from the Sun to Earth is 150 000 000 km. What is the average distance between Earth and Mars?

Solution

Step 1: If you find the large numbers overwhelming, make up a related but simpler problem.

Sam and Teresa both live east of the store. Sam lives 3 km from the store and Teresa lives 6 km from the store. How far is it from Sam's house to Teresa's house?



$6 - 3 = 3$

It is 3 km from Sam's house to Teresa's house.

Step 2: Once you understand the related but simpler problem, you can solve the original problem.

$$228\,000\,000 - 150\,000\,000 = 78\,000\,000$$

The average distance from Earth to Mars is 78 000 000 km.

Finding and Applying a Pattern

To solve some problems, it is often helpful to use a related but simpler problem to discover a pattern.

Example

There are 20 students in a room. If every student shakes hands with every other student in the room, how many handshakes are exchanged?

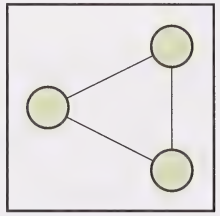
Solution

Step 1: Try solving the problem with 1, 2, 3, 4, or 5 students.

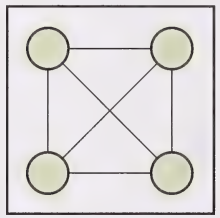
- If there is 1 student, no handshakes can be exchanged (as illustrated in the following diagram).

- If there are 2 students, 1 handshake can be exchanged (as illustrated in the following diagram).

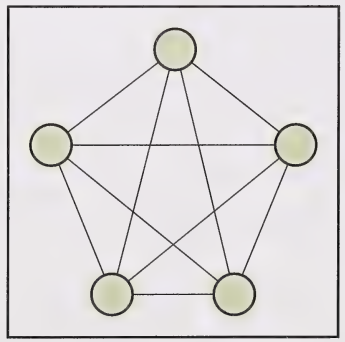
- If there are 3 students, 3 handshakes can be exchanged (as illustrated in the following diagram).



- If there are 4 students, 6 handshakes can be exchanged (as illustrated in the following diagram).



- If there are 5 students, 10 handshakes can be exchanged (as illustrated in the following diagram).

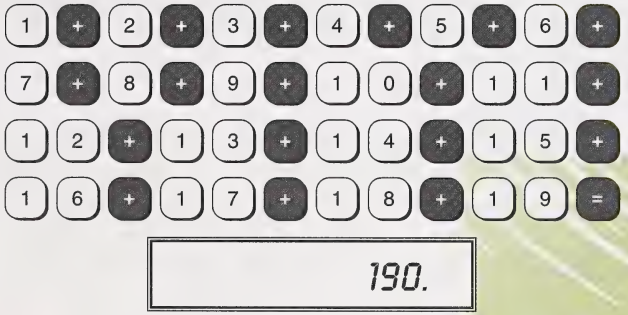


Step 2: Look for a pattern.

People	Handshakes	Pattern
1	0	
2	1	+1
3	3	+2
4	6	+3
5	10	+4

Step 3: Apply the pattern.

A calculator is helpful for applying the pattern. Go up to 19, because each student will shake the hand of 19 others.

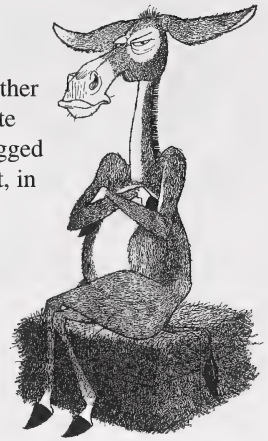


There are a total of 190 handshakes exchanged.

Using Elimination

Detectives are skilful at eliminating clues and solving mysteries. You can become skilful at using elimination to solve problems too.

Example



Five animals had a race over a short distance. Neither the donkey nor the coyote beat the lion. The coyote beat the rabbit, but not the ostrich. The donkey lagged behind the rabbit. If the ostrich was not the fastest, in what order did the animals finish the race?

Solution

Step 1: Decide what clues are given. The clues indicate the following:

- The donkey and the coyote did not finish first because they did not beat the lion.
- The rabbit did not finish first because the coyote beat it.
- The ostrich was faster than the coyote.
- The rabbit was faster than the donkey.
- The ostrich did not finish first because it was not the fastest.

The clues can be shown on a table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	
Second					
Third					
Fourth					
Fifth					

Step 2: Use elimination and reconsider the clues.

By elimination, it is clear that the lion was first. Now, reconsider the clues:

- The rabbit did not finish second because the coyote beat it.
- The coyote did not finish second because the ostrich was faster.
- The donkey did not finish second because it lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	
Second	✗	✗	✗		✗
Third					
Fourth					
Fifth					

Step 3: Use elimination and reconsider the clues.

By elimination, it is clear that the ostrich finished second. Now, reconsider the clues:

- The rabbit did not finish third because the coyote beat it.
- The donkey did not finish third because it lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗		✗	✗	✗
Fourth				✗	✗
Fifth				✗	✗

Step 4: Use elimination and reconsider the clues.

By elimination, it is clear that the coyote finished third. Now, reconsider the clues.

Because the donkey lagged behind the rabbit, the rabbit was fourth and the donkey was fifth.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗	✓	✗	✗	✗
Fourth	✗	✗	✓	✗	✗
Fifth	✓	✗	✗	✗	✗

So, the lion was first, the ostrich was second, the coyote was third, the rabbit was fourth, and the donkey was last.

Using Truth Tables

A strategy you can use to solve some logic problems is making truth tables. Truth tables are similar to elimination tables.

Example

Detective Beagle is investigating a robbery in the forest. Charlie Chipmunk and Sammy Squirrel each deny being the robber. Harold Hare blames Charlie Chipmunk. Detective Beagle knows two of the animals are lying and one is telling the truth. Who is the robber?

Solution

Step 1: Assume that Charlie Chipmunk was the robber.

- Charlie Chipmunk said he didn't do it. If Charlie Chipmunk was the robber, this is a false statement. Make a truth table and put **F** under Charlie's name.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F		

- Sammy Squirrel denied doing it. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	**T**	

- Harold Hare blamed Charlie Chipmunk. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	**T**

Detective Beagle knows two animals are lying. So, Charlie is not the robber.

Step 2: Assume that Sammy Squirrel was the robber.

- Charlie Chipmunk denied doing it. If Sammy Squirrel was the robber, this is a true statement. Put **T** under Charlie's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T		

- Sammy Squirrel denied doing it. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	**F**	

- Harold Hare blamed Charlie Chipmunk. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	F	**F**

Detective Beagle knows two of the animals are lying and one is telling the truth.

So, Sammy Squirrel was the robber.

Using an Equation

A strategy you can use to solve some problems is to write and solve an equation.

Example

Twelve less than a number is -3. Find the number.

Solution

Step 1: Let the unknown be equal to a variable. Let the number be n.

Step 2: Write an equation.

$$n - 12 = -3$$

Twelve less than a number is -3.

Step 3: Solve the equation.

$$\begin{array}{r} n - 12 = -3 \\ +12 \quad +12 \\ \hline n \quad = \quad 9 \end{array}$$

The number is 9.

Example

The second of two numbers is 7 times the first. The sum of the two numbers is 32. Find the numbers.

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let n be the first number and let $7n$ be the second number.

The second of two numbers is 7 times the first.

Step 2: Write an equation.

$$n + 7n = 32$$

The sum of the two numbers is 32.

Step 3: Solve the equation.

$$\begin{aligned} n + 7n &= 32 \\ 8n &= 32 \\ \frac{8n}{8} &= \frac{32}{8} \\ n &= 4 \end{aligned}$$

The first number is 4.

Step 4: Find the second number if the first number is 4.

$$\begin{aligned} 7n &= 7 \times 4 \\ &= 28 \end{aligned}$$

The second number is 28.

So, the numbers are 4 and 28.

Example

Yvonne has 20 more nickels than dimes in her piggy bank. If the total value of the nickels and dimes is $8.50, how many coins of each type does she have?

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let the number of dimes be n.

The value of the nickels, the number of dimes, and the value of the dimes must also be represented by algebraic expressions.

A table can be used to organize the information.

Type of Coin	Number of Coins	Value in Cents
Dimes	n	$10n$
Nickels	$n + 20$	$5(n + 20)$
Total		850

The value of the dimes is 10 times the number of dimes.

The value of the nickels is 5 times the number of nickels.

Yvonne has 20 more nickels than dimes.

This is given.

Step 2: Write an equation.

$$5(n+20)+10n=850$$

Step 3: Solve the equation.

$$\begin{aligned} 5(n+20)+10n &= 850 \\ 5n+100+10n &= 850 \\ 15n+100 &= 850 \\ -100 \quad &-100 \\ \hline 15n \quad &= 750 \\ \frac{15n}{15} &= \frac{750}{15} \\ n &= 50 \end{aligned}$$

Yvonne has 50 dimes in her piggy bank.

Step 4: Find the number of nickels if the number of dimes is 50.

$$\begin{aligned} n+20 &= 50+20 \\ &= 70 \end{aligned}$$

Yvonne has 70 nickels in her piggy bank.

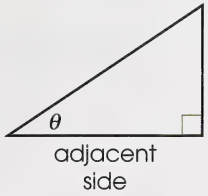
So, Yvonne has 50 dimes and 70 nickels in her piggy bank.

Glossary

acute angle: an angle that is less than a quarter turn; an angle less than 90°

adjacent angles: any two angles that lie on opposite sides of a common arm

adjacent side: the leg next to the reference angle



angle: a geometric figure formed by two distinct rays with the same endpoint

angle of depression: the angle through which an observer looks downward from the horizontal to see an object below

angle of elevation: the angle through which an observer looks upward from the horizontal to see an object above

complementary angles: two angles that have a sum of 90°

congruent figures: figures that have the same size and shape

cosine: the ratio formed by dividing the length of the side adjacent to the reference angle by the length of the hypotenuse

dilatation: a transformation that enlarges or reduces a figure without changing its shape

exponential form: a number written as a power

The exponential form of 81 is 9^2 or 3^4.

hypotenuse: the side opposite the right angle in a right triangle; the longest side

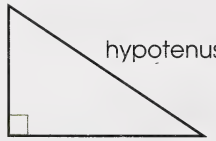


image: the resultant figure after performing a transformation on a figure

integers: the set of numbers that consists of the whole numbers and their opposites

$I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

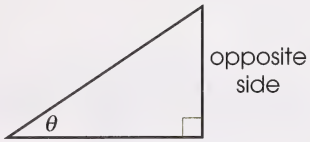
irrational number: a number that cannot be written in the form $\frac{a}{b}$, where a and b are both integers

mapping rule: directions for finding the image of a figure

e.g., $(x, y) \rightarrow (2x, 2y)$ ← indicates that each point of the image has coordinates that are double the coordinates of the points on the original figure

obtuse angle: an angle that is more than a quarter turn and less than half a turn; an angle between 90° and 180°

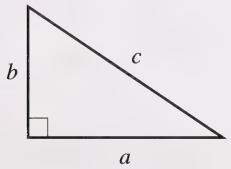
opposite side: the leg opposite the reference angle



perfect square: a number that has a natural number as its principal square root

Because $\sqrt{100} = 10$, 100 is a perfect square.

Pythagorean Theorem: a theorem stating that the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse



$c^2 = a^2 + b^2$

Pythagorean triple: three numbers whose sum of the squares of the two smaller numbers equals the square of the largest number

ratio: a comparison of quantities measured in the same unit

rational number: a number that can be written as a fraction of the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$

reflex angle: an angle that measures more than half a turn and less than a full turn; an angle between 180° and 360°

right angle: an angle that measures exactly a quarter turn; a 90° angle

right triangle: a triangle that has one angle measuring 90°

scale drawing: a drawing larger or smaller than actual size used to portray a person, animal, or thing that is too large or too small to be drawn to actual size

scale factor: the ratio of the lengths of the sides of the image to the lengths of the sides of the original figure

e.g., $(x, y) \rightarrow (ax, ay)$ — a: scale factor

scale model: a likeness of an object that has the same proportions as the object

similar figures: figures that have the same shape

sine: the ratio formed by dividing the length of the side opposite the reference angle by the length of the hypotenuse

square root: one of two equal factors that yields a product equal to a given number

straight angle: an angle that is exactly a half turn; a 180° angle

supplementary angles: two angles that have a sum of 180°

tangent: the ratio formed by dividing the length of the side opposite the reference angle by the length of the side adjacent to the reference angle

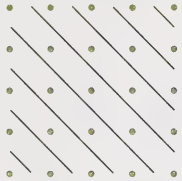
transversal: a line intersecting two or more other lines

trigonometry: the study of the relationships of angles and sides in right triangles

Suggested Answers

Section 1: Activity 1

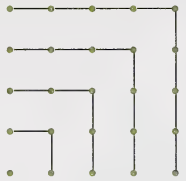
1. a.



$5^2 = 1+2+3+4+5+4+3+2+1$

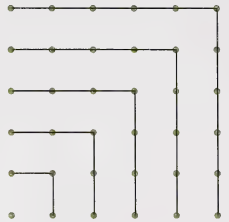
b. $10^2 = 1+2+3+4+5+6+7+8+9+10+9+8+7+6+5$
$+4+3+2+1$

2. a.



$5^2 = 1+3+5+7+9$

b.



$6^2 = 1+3+5+7+9+11$

c. $10^2 = 1+3+5+7+9+11+13+15+17+19$

3. a.

Distance from Screen (units)	Area of Picture (square units)
1	1
2	4
3	9
4	16

b. $a = d^2$

c. $a = d^2$
$= 5^2$
$= 25$

If the projector is 5 units from the screen, the area of the picture on the screen is 25 square units.

4. $d = 5t^2$

$= 5(4)^2$

$= 5(16)$

$= 80$

In the first 4 s, the object will fall 80 m.

5. a. $d = \frac{s^2}{210}$

$= \frac{50^2}{210}$

$= \frac{2500}{210}$

$\doteq 11.9$

The braking distance is is about 11.9 m.

b. $d = \frac{s^2}{210}$

$= \frac{100^2}{210}$

$= \frac{10\,000}{210}$

$\doteq 47.6$

The braking distance is about 47.6 m.

6. a. $\sqrt{100} = 10$ **b.** $\sqrt{256} = 16$ **c.** $-\sqrt{81} = -9$

d. $-\sqrt{400} = -20$ **e.** $\pm\sqrt{121} = \pm 11$ **f.** $\pm\sqrt{144} = \pm 12$

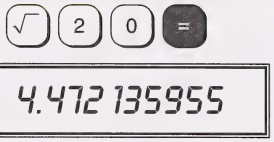
7. a. **Estimate**

$\sqrt{16} < \sqrt{20} < \sqrt{25}$

$4 < \sqrt{20} < 5$

Because 20 is approximately halfway between 16 and 25, $\sqrt{20} \doteq 4.5$.

Decimal Approximation



$\therefore \sqrt{20} \doteq 4.47$

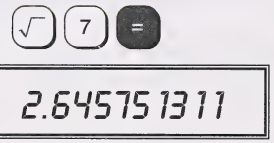
b. **Estimate**

$\sqrt{4} < \sqrt{7} < \sqrt{9}$

$2 < \sqrt{7} < 3$

Because 7 is approximately halfway between 4 and 9, $\sqrt{7} \doteq 2.5$.

Decimal Approximation



$\therefore \sqrt{7} \doteq 2.65$

c. **Estimate**

Round 1215 to the nearest 100.

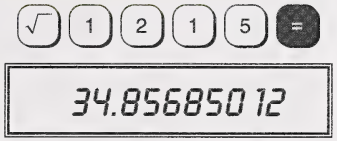
$1215 \doteq 1200$

$\therefore \sqrt{900} < \sqrt{1200} < \sqrt{1600}$

$30 < \sqrt{1200} < 40$

Because 1200 is approximately halfway between 900 and 1600, $\sqrt{1215} \doteq 35$.

Decimal Approximation



$\therefore \sqrt{1215} \doteq 34.86$

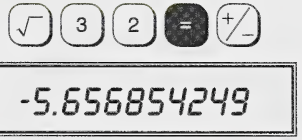
d. **Estimate**

$-\sqrt{36} < -\sqrt{32} < -\sqrt{25}$

$-6 < -\sqrt{32} < -5$

Because 32 is approximately halfway between 25 and 36, $-\sqrt{32} \doteq -5.5$.

Decimal Approximation



$\therefore -\sqrt{32} \doteq -5.66$

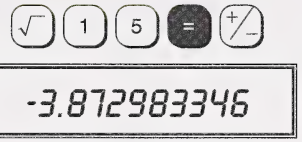
e. **Estimate**

$-\sqrt{16} < -\sqrt{15} < -\sqrt{9}$

$-4 < -\sqrt{15} < -3$

Because 15 is close to 16, $-\sqrt{15} \doteq -4$.

Decimal Approximation



$\therefore -\sqrt{15} \doteq -3.87$

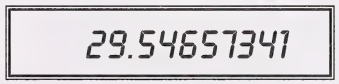
f. Estimate

Round 873 to the nearest 100.

$873 \doteq 900$

$\therefore \sqrt{873} \doteq 30$

Decimal Approximation



$\therefore \sqrt{873} \doteq 29.55$

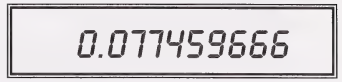
g. Estimate

$\sqrt{0.0049} < \sqrt{0.0060} < \sqrt{0.0064}$

$0.07 < \sqrt{0.0060} < 0.08$

Because 0.0060 is closer to 0.0064, $\pm\sqrt{0.0064} \doteq \pm 0.08$.

Decimal Approximation



$\therefore \pm\sqrt{0.006} \doteq \pm 0.08$

h. Estimate

Round 23 815 to the nearest 100.

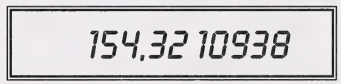
$23\,815 \doteq 23\,800$

$\sqrt{22\,500} < \sqrt{23\,800} < \sqrt{25\,600}$

$150 < \sqrt{23\,800} < 160$

Because 23 800 is approximately halfway between 22 500 and 25 000, $\pm\sqrt{23\,815} \doteq \pm 155$.

Decimal Approximation



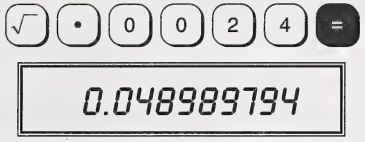
$\therefore \pm\sqrt{23\,815} \doteq \pm 154.32$

i. Estimate

$$\sqrt{0.0016} < \sqrt{0.0024} < \sqrt{0.0025}$$
$$0.04 < \sqrt{0.0024} < 0.05$$

Because 0.0024 is close to 0.0025, $\pm\sqrt{0.0025} \doteq \pm 0.05$.

Decimal Approximation



$\therefore\ \pm\sqrt{0.0024} \doteq \pm 0.05$

8. a. The digits filled the entire display for every answer.

b. Yes, the calculator would display as many digits as possible. The square root of a number that is not a perfect square is a non-terminating, non-repeating decimal.

Looking Back

9. 1, 8, 27, **64**, **125**, **216**, **343**

10. a. Because **100** lies between **64** and **125**, $\sqrt[3]{100}$ lies between **4** and **5** and is closer to **5**.

b. Because **36** lies between **27** and **64**; so, $\sqrt[3]{36}$ lies between **3** and **4** and is closer to **3**.

Section 1: Activity 2

1. a.
$$x^2 + 1 = 10$$
$$x^2 + 1 - 1 = 10 - 1$$
$$x^2 = 9$$
$$x = \sqrt{9} \text{ or } -\sqrt{9}$$
$$= 3 \text{ or } -3$$

b.
$$r^2 - 1 = 24$$
$$r^2 - 1 + 1 = 24 + 1$$
$$r^2 = 25$$
$$r = \sqrt{25} \text{ or } -\sqrt{25}$$
$$= 5 \text{ or } -5$$

c.
$$3t^2 = 108$$
$$\frac{3t^2}{3} = \frac{108}{3}$$
$$t^2 = 36$$
$$t = \sqrt{36} \text{ or } -\sqrt{36}$$
$$= 6 \text{ or } -6$$

d.
$$\frac{n^2}{2} = 32$$
$$n^2 = 64$$
$$n = \sqrt{64} \text{ or } -\sqrt{64}$$
$$= 8 \text{ or } -8$$

e.
$$25x^2 + 1 = 65$$
$$25x^2 + 1 - 1 = 65 - 1$$
$$25x^2 = 64$$
$$\frac{25x^2}{25} = \frac{64}{25}$$
$$x^2 = \frac{64}{25}$$
$$x = \sqrt{\frac{64}{25}} \text{ or } -\sqrt{\frac{64}{25}}$$
$$= \frac{8}{5} \text{ or } -\frac{8}{5}$$

f.
$$4b^2 - 3 = 33$$
$$4b^2 - 3 + 3 = 33 + 3$$
$$4b^2 = 36$$
$$\frac{4b^2}{4} = \frac{36}{4}$$
$$b^2 = 9$$
$$b = \sqrt{9} \text{ or } -\sqrt{9}$$
$$= 3 \text{ or } -3$$

2. Let n represent the number.

a.
$$n^2 - 8 = 17$$
$$n^2 - 8 + 8 = 17 + 8$$
$$n^2 = 25$$
$$n = \sqrt{25} \text{ or } -\sqrt{25}$$
$$= 5 \text{ or } -5$$

The number is 5 or −5.

b.
$$10n^2 = 40$$
$$\frac{10n^2}{10} = \frac{40}{10}$$
$$n^2 = 4$$
$$n = \sqrt{4} \text{ or } -\sqrt{4}$$
$$= 2 \text{ or } -2$$

The number is 2 or −2.

3. $\ell = 82$ and $g = 50$

$$\therefore m = \frac{\ell \times g^2}{24\,600}$$
$$= \frac{82 \times 50^2}{24\,600}$$
$$= \frac{82 \times 2500}{24\,600}$$
$$\doteq 8.3$$

The mass of the salmon is approximately 8.3 kg.

4. a.
$$d = \frac{v^2}{210}$$
$$= \frac{80^2}{210}$$
$$= \frac{6400}{210}$$
$$\doteq 30.5$$

The breaking distance is about 30.5 m.

b.
$$d = \frac{v^2}{210}$$
$$62 = \frac{v^2}{210}$$
$$210(62) = \cancel{210}\left(\frac{v^2}{\cancel{210}}\right)$$
$$13\,020 = v^2$$
$$v = \sqrt{13\,020}$$
$$\doteq 114$$

The speed of the vehicle was about 114 km/h.

5. **Estimate**

$$A = \pi r^2$$
$$153.9 = \pi r^2$$
$$150 \doteq 3r^2$$
$$\frac{150}{3} \doteq r^2$$
$$50 \doteq r^2$$
$$r \doteq \sqrt{50}$$
$$\doteq 7$$

The radius of the circle is about 7 cm.

Decimal Approximation

$$A = \pi r^2$$
$$153.9 = \pi r^2$$
$$\frac{153.9}{\pi} = r^2$$
$$r = \sqrt{\frac{153.9}{\pi}}$$
$$\doteq 7$$

6. a. Only the positive value of the radius was used because radius is a length and cannot be negative.

b. The positive value of a square root is called the principal square root.

c. No. A calculator only gives the positive value for the square root of a positive number.

d. The calculator gives an error message when you try to take the square root of a negative number. This is because there is no real number that, when multiplied by itself, gives a negative number.

7. **Estimate**

Estimate the area of the cross-stitch picture.

$$\begin{aligned} A &= s^2 \\ &= 24^2 \\ &\doteq 25^2 \\ &\doteq 625 \text{ cm}^2 \end{aligned}$$

Estimate the area of the drymount board.

$$\begin{aligned} A &\doteq 2 \times 625 \\ &\doteq 1250 \text{ cm}^2 \end{aligned}$$

Estimate the length of each side of the drymount board.

$$\begin{aligned} s &= \sqrt{A} \\ &= \sqrt{1250} \\ &\doteq 35 \text{ cm} \end{aligned}$$

$$\sqrt{900} < \sqrt{1250} < \sqrt{1600}$$
$$30 < \sqrt{1250} < 40$$

Decimal Approximation

Determine the area of the cross-stitch picture.

$$\begin{aligned} A &= s^2 \\ &= 24^2 \\ &= 576 \text{ cm}^2 \end{aligned}$$

Determine the area of drymount board.

$$\begin{aligned} A &= 2 \times 576 \\ &= 1152 \text{ cm}^2 \end{aligned}$$

Calculate the length of each side of the drymount board.

$$\begin{aligned} s &= \sqrt{A} \\ &= \sqrt{1152} \\ &\doteq 34 \end{aligned}$$

The length of each side of the drymount board is approximately 34 cm.

8. Find the area of the pasture.

$$A = 4 \times 10\ 000$$
$$= 40\ 000\ \text{m}^2$$

Find the length of each side of the pasture.

$$\ell = \sqrt{A}$$
$$= \sqrt{40\ 000}$$
$$= 200\ \text{m}$$

Therefore, the pasture requires $200 \times 4 = 800$ m of fencing.

9. You can find the length of each side of the square by determining the square root of the area. Because the perimeter of a square is four times the length of one side, multiply the length by 4 to calculate the perimeter.

Looking Back

10. a.
$$3x^2 + 87 = 162$$
$$3x^2 + 87 - 87 = 162 - 87$$
$$3x^2 = 75$$
$$\frac{3x^2}{3} = \frac{75}{3}$$
$$x^2 = 25$$
$$x = \pm\sqrt{25}$$
$$= \pm 5$$

b.
$$V = \pi r^2 h$$
$$340 \doteq \pi r^2 (12)$$
$$\frac{340}{\pi(12)} \doteq \frac{\pi r^2 (12)}{\pi(12)}$$
$$\frac{340}{12\pi} = r^2$$
$$r \doteq \sqrt{\frac{340}{12\pi}}$$
$$\doteq 3.0$$

The radius is approximately 3.0 cm.

Section 1: Activity 3

1. a. Yes, Square B and the four pieces of Square A cover Square C.

b. The sum of the squares of the legs equals the square of the hypotenuse; that is, $a^2 + b^2 = c^2$.

2. a. Yes, Square B and the four pieces of Square A cover Square C.

b. The sum of the squares of the legs equals the square of the hypotenuse; that is, $a^2 + b^2 = c^2$.

3. a. Write the formula for the Pythagorean Theorem, and substitute the given values. Then simplify the equation.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 35^2 + 12^2 &= c^2 \\ 1225 + 144 &= c^2 \\ 1369 &= c^2 \\ c &= \sqrt{1369} \\ &= 37 \text{ cm} \end{aligned}$$

b.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 30^2 + 16^2 &= c^2 \\ 900 + 256 &= c^2 \\ 1156 &= c^2 \\ c &= \sqrt{1156} \\ &= 34 \text{ mm} \end{aligned}$$

c.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 55^2 + b^2 &= 73^2 \\ 3025 + b^2 &= 5329 \\ 3025 + b^2 - 3025 &= 5329 - 3025 \\ b^2 &= 2304 \\ b &= \sqrt{2304} \\ &= 48 \text{ cm} \end{aligned}$$

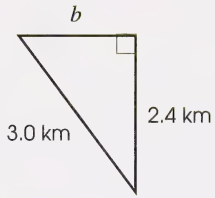
d.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6.3^2 + b^2 &= 6.5^2 \\ 39.69 + b^2 &= 42.25 \\ 39.69 + b^2 - 39.69 &= 42.25 - 39.69 \\ b^2 &= 2.56 \\ b &= \sqrt{2.56} \\ &= 1.6 \text{ m} \end{aligned}$$

e.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5.2^2 + 2.0^2 &= c^2 \\ 27.04 + 4 &= c^2 \\ 31.04 &= c^2 \\ c &= \sqrt{31.04} \\ &\doteq 5.6 \text{ km} \end{aligned}$$

f.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 2.0^2 &= 2.9^2 \\ a^2 + 4 &= 8.41 \\ a^2 + 4 - 4 &= 8.41 - 4 \\ a^2 &= 4.41 \\ a &= \sqrt{4.41} \\ &= 2.1 \text{ cm} \end{aligned}$$

4.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 2.4^2 + b^2 &= 3.0^2 \\ 5.76 + b^2 &= 9 \\ 5.76 + b^2 - 5.76 &= 9 - 5.76 \\ b^2 &= 3.24 \\ b &= \sqrt{3.24} \\ &= 1.8 \end{aligned}$$

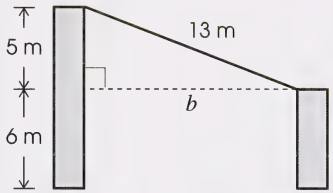


Geeta's cabin is 1.8 km from Jack's cabin.

Remember: Measurements are always positive; therefore, only use the positive (or principal) square root.

5.

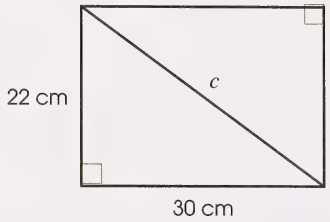
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + b^2 &= 13^2 \\ 25 + b^2 &= 169 \\ 25 + b^2 - 25 &= 169 - 25 \\ b^2 &= 144 \\ b &= \sqrt{144} \\ &= 12 \end{aligned}$$



The distance between the walls is 12 m.

6.

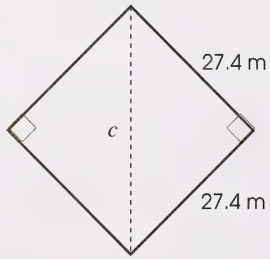
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 22^2 + 30^2 &= c^2 \\ 484 + 900 &= c^2 \\ 1384 &= c^2 \\ c &= \sqrt{1384} \\ &\doteq 37.2 \end{aligned}$$



The diagonal measure of the screen is about 37.2 cm.

7.

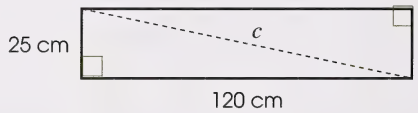
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 27.4^2 + 27.4^2 &\doteq c^2 \\ 750.76 + 750.76 &\doteq c^2 \\ 1501.52 &\doteq c^2 \\ c &\doteq \sqrt{1501.52} \\ &\doteq 38.7 \end{aligned}$$



The player must throw the ball about 38.7 m.

8.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 25^2 + 120^2 &= c^2 \\ 625 + 14\,400 &= c^2 \\ 15\,025 &= c^2 \\ c &= \sqrt{15\,025} \\ &\doteq 122.6 \end{aligned}$$



The longest ski pole that could be packed to lie flat is about 122.6 cm.

Did You Know?

9. a. $5^2 = 25 \quad 7^2 = 49 \quad 10^2 = 100$

$25 + 49 \neq 100$

No, 5, 7, and 10 is not a Pythagorean triple.

b. $7^2 = 49 \quad 24^2 = 576 \quad 25^2 = 625$

$49 + 576 = 625$

Yes, 7, 24, and 25 is a Pythagorean triple.

c. $9^2 = 81 \quad 13^2 = 169 \quad 14^2 = 196$

$81 + 169 \neq 196$

No, 9, 13, and 14 is not a Pythagorean triple.

d. $5^2 = 25 \quad 12^2 = 144 \quad 13^2 = 169$

$25 + 144 = 169$

Yes, 5, 12, and 13 is a Pythagorean triple.

e. $7^2 = 49 \quad 20^2 = 400 \quad 21^2 = 441$

$49 + 400 \neq 441$

No, 7, 20, and 21 is not a Pythagorean triple.

f. $1^2 = 1 \quad 10^2 = 100 \quad 11^2 = 121$

$1 + 100 \neq 121$

No, 1, 10, and 11 is not a Pythagorean triple.

10. a. $6^2 = 36 \quad 8^2 = 64 \quad 10^2 = 100$

$36 + 64 = 100$

Yes, 6, 8, and 10 is a Pythagorean triple.

b. $9^2 = 81 \quad 12^2 = 144 \quad 15^2 = 225$

$81 + 144 = 225$

Yes, 9, 12, and 15 is a Pythagorean triple.

c. Answers may vary. Sample Pythagorean triples are given.

$3 \times 4 = 12$	$3 \times 5 = 15$	$3 \times 6 = 18$
$4 \times 4 = 16$	$4 \times 5 = 20$	$4 \times 6 = 24$
$5 \times 4 = 20$	$5 \times 5 = 25$	$5 \times 6 = 30$

11. Once you know a Pythagorean triple, you can find others by multiplying each number in the triple by the same factor.

Looking Back

12. The Pythagorean Theorem is a theorem stating that the sum of the squares of each leg of a right triangle is equal to the square of the hypotenuse.

$$a^2 + b^2 = c^2$$

Section 2: Activity 1

1. a. obtuse angle **b.** right angle **c.** acute angle

d. straight angle **e.** acute angle **f.** right angle

2. a. $\angle CED$ and $\angle DEF$ are **complementary** angles.

b. $\angle RVS$ and $\angle TVU$ are **complementary** angles.

c. $\angle AEB$ and $\angle AEC$ are **supplementary** angles.

d. $\angle YVZ$ and $\angle WVX$ are **supplementary** angles.

3. a. $x + 75° = 90°$ ← The sum of complementary angles is 90°.

$x = 15°$

b. $30° + 50° + r = 90°$ ← The sum of complementary angles is 90°.

$80° + r = 90°$

$r = 10°$

4. a. $m + 65° = 180°$ ← The sum of supplementary angles is 180°.

$m = 115°$

b. $45° + 58° + b = 180°$ ← The sum of supplementary angles is 180°.

$103° + b = 180°$

$b = 77°$

5. Yes, the three angles of each triangle form a straight angle.

6. a. $23° + 120° + c = 180°$ ← The sum of the angles of a triangle is 180°.

$143° + c = 180°$

$c = 37°$

b. $w + 90° + 45° = 180°$ ← The sum of the angles of a triangle is 180°.

$w + 135° = 180°$

$w = 45°$

c. $r + 45° + 63° = 180°$ ← The sum of the angles of a triangle is 180°.

$r + 108° = 180°$

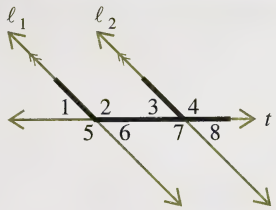
$r = 72°$

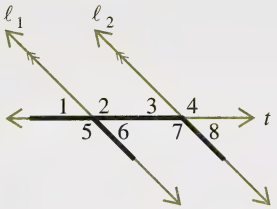
d. $x + 135° + 25° = 180°$ ← The sum of the angles of a triangle is 180°.

$x + 160° = 180°$

$x = 20°$

7. a.





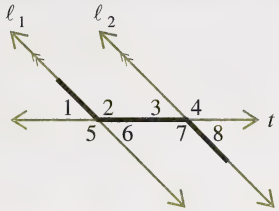
$\angle 1$ and $\angle 3$ are corresponding angles; $\angle 1 = \angle 3$.

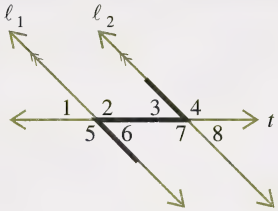
$\angle 2$ and $\angle 4$ are corresponding angles; $\angle 2 = \angle 4$.

$\angle 5$ and $\angle 7$ are corresponding angles; $\angle 5 = \angle 7$.

$\angle 6$ and $\angle 8$ are corresponding angles; $\angle 6 = \angle 8$.

b.





$\angle 1$ and $\angle 8$ are exterior alternate angles; $\angle 1 = \angle 8$.

$\angle 2$ and $\angle 7$ are interior alternate angles; $\angle 2 = \angle 7$.

$\angle 3$ and $\angle 6$ are interior alternate angles; $\angle 3 = \angle 6$.

$\angle 4$ and $\angle 5$ are exterior alternate angles; $\angle 4 = \angle 5$.

8. a. $\angle 5$ and $\angle 7$ are **corresponding** angles.

b. $\angle 5$ and $\angle 4$ are **exterior alternate** angles.

c. $\angle 2$ and $\angle 7$ are **interior alternate** angles.

d. $\angle 2$ and $\angle 4$ are **corresponding** angles.

9. The angle of elevation and the angle of depression are equal. Because the horizontal drawn from the balloon is parallel to the horizontal drawn from your eye level, the angle of depression and the angle of elevation are interior alternate angles formed by the line of sight. For parallel lines cut by a transversal, interior alternate angles are equal.

Looking Back

10. The sum of the angles in a triangle is **180°**.

An acute angle is less than **90°**, a **right** angle is exactly 90°, and an **obtuse** angle is between 90° and 180°.

A **straight** angle is exactly 180°, and an angle between 180° and 360° is a **reflex** angle.

If two angles form a straight angle, they are **supplementary** angles. If two angles form a right angle, they are **complementary** angles.

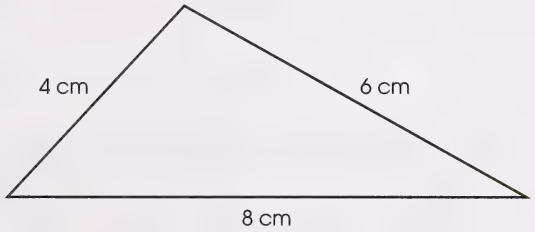
Section 2: Activity 2

1. Yes, the two figures coincide exactly.

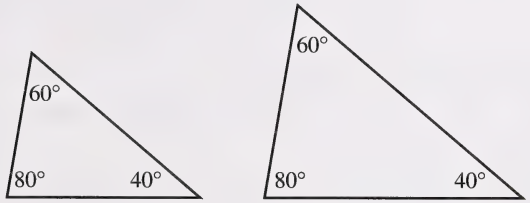
2. Yes, the two figures appear to be the same size and shape.

3. You would get four figures that appear to be the same size and shape.

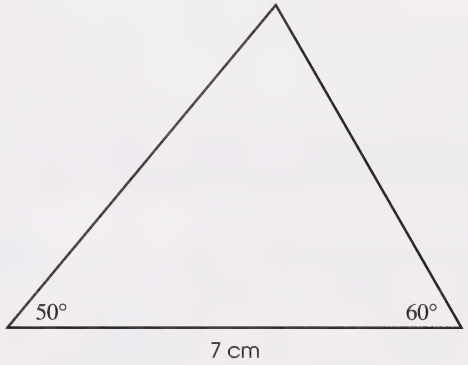
4. **Property 1:** The triangles will all look like this.



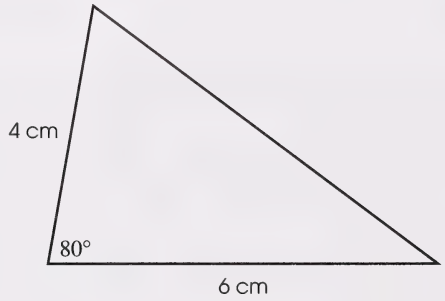
Property 2: The triangles will all be similar in shape, but may be different in size.



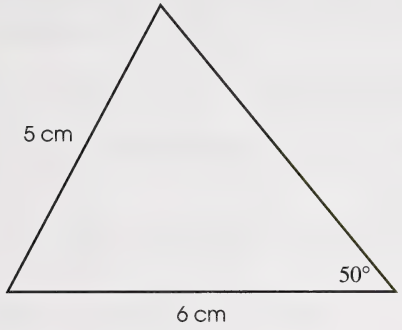
Property 3: The triangles will all look like this.

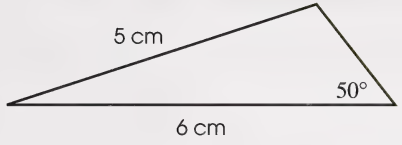


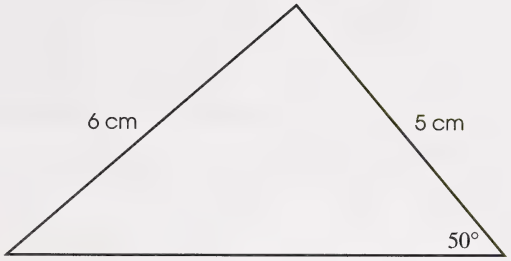
Property 4: The triangles will all look like this.



Property 5: The triangles will look like any of these.







5. **Property 1:** Yes
Property 2: No
Property 3: Yes
Property 4: Yes
Property 5: No

6. Two triangles are congruent under the following conditions:

- the three pairs of corresponding sides are equal in the two triangles
- two angles and one side of a triangle are equal to the corresponding two angles and side of a second triangle
- two sides and the contained angle of a triangle are equal to the corresponding two sides and contained angle of a second triangle

7. **a.** SSS
$\Delta ABC \cong \Delta XYZ$

b. SAS
$\Delta CAT \cong \Delta DOG$

8. **a.** $\angle A = \angle X$ $\quad AB = XY$
$\angle B = \angle Y$ $\quad BC = YZ$
$\angle C = \angle Z$ $\quad AC = XZ$

b. $\angle C = \angle D$ $\quad CA = DO$
$\angle A = \angle O$ $\quad AT = OG$
$\angle T = \angle G$ $\quad CT = DG$

9. a. $\angle T = \angle N$ or $\angle O = \angle A$ or $PO = MA$

b. $UY = WO$ or $BY = NO$ or $BU = NW$

c. $NT = PN$ or $\angle O = \angle A$

10. a.

$BC = 32$ cm	$\angle C = 42°$
$DF = 12$ cm	$\angle B = 18°$
$DE = 24$ cm	$\angle E = 18°$

b.

$BO = 2.4$ cm	$\angle Y = 37°$
$OY = 3.5$ cm	$\angle M = 53°$
$MN = 4.2$ cm	$\angle A = 90°$
	$\angle N = 37°$

11. a. $\angle ACB = \angle ACD$ because the other two angles in the two triangles are equal. That is, $\angle ACB = 180° - (x + y) = \angle ACD$, if $\angle BAC = \angle DAC = x$ and $\angle ABC = \angle ADC = y$.

b. Yes, AC is common to both triangles; so, $\Delta ABC \cong \Delta ADC$ (ASA or SAA).

12. a. Two triangles with the same perimeter are sometimes congruent.

b. Two triangles with the same area are sometimes congruent.

c. Two squares with the same perimeter are always congruent. Two squares with the same area are always congruent. This is true because the angles in a square are all right angles and the length of the sides are determined by the perimeter or the area of the square.

Looking Back

13. a. Use the Pythagorean Theorem to calculate MN.

$$\begin{aligned} (MN)^2 + (NO)^2 &= (MO)^2 \\ (MN)^2 + 12^2 &= 13^2 \\ (MN)^2 + 144 &= 169 \\ (MN)^2 &= 25 \\ MN &= \sqrt{25} \\ &= 5 \end{aligned}$$

b. Because ΔMNO and ΔQRS are right-angle triangles and both MN and QR have a length of 5, the triangles are congruent according to the SSS condition.

Section 2: Activity 3

1. $\angle A = \angle R = 37°$
$\angle B = \angle S = 53°$
$\angle C = \angle T = 90°$

The corresponding angles are equal.

2. a. i. $\frac{AC}{AB} = \frac{4}{5}$ $\quad \frac{RT}{RS} = \frac{8}{10}$

$= \frac{4}{5}$

ii. $\frac{BC}{AB} = \frac{3}{5}$ $\quad \frac{ST}{RS} = \frac{6}{10}$

$= \frac{3}{5}$

iii. $\frac{BC}{AC} = \frac{3}{4}$ $\quad \frac{ST}{RT} = \frac{6}{8}$

$= \frac{3}{4}$

b. The ratio of any two sides of ΔABC are the same as the ratio of the two corresponding sides of ΔRST.

3. a. $\frac{DE}{XY} = \frac{6}{9}$ $\quad \frac{DF}{XZ} = \frac{3}{4.5}$ $\quad \frac{EF}{YZ} = \frac{5}{7.5}$

$= \frac{2}{3}$ $\quad = \frac{2}{3}$ $\quad = \frac{2}{3}$

The ratios of pairs of corresponding sides are equal.

b. i. $\frac{DE}{DF} = \frac{6}{3}$ $\quad \frac{XY}{XZ} = \frac{9}{4.5}$

$= \frac{2}{1}$ $\quad = \frac{2}{1}$

ii. $\frac{DF}{EF} = \frac{3}{5}$ $\quad \frac{XZ}{YZ} = \frac{4.5}{7.5}$

$= \frac{3}{5}$

4. The ratios are equal because you are comparing ratios of pairs of corresponding sides in the two similar triangles.

5. a. $180 - (40 + 60) = 80°$

$\therefore \angle C = 80°$ and $\angle F = 80°$

b. Yes, $\Delta ABC \sim \Delta DEF$ because their corresponding angles have the same measure.

6. a. $\frac{24}{36} = \frac{30}{45} = \frac{52}{78}$ ← Each fraction equals $\frac{2}{3}$.

Yes, these triangles are similar since the corresponding sides are proportional.

b. Not enough information is given to determine if these triangles are similar.

c. Yes, these triangles are similar because the third angle of both triangles is $180° - (53° + 75°) = 52°$; so, the corresponding angles are equal.

d. $\frac{5}{2.2} \neq \frac{3.5}{2.2}$

All the sides of one triangle are not proportional to the corresponding sides of the other triangle. One triangle is isosceles (two equal sides), and the other triangle is equilateral (three equal sides).

7. a. The following is one way to solve this question.

$$\frac{BC}{EF} = \frac{AB}{DE} \qquad \frac{AC}{DF} = \frac{AB}{DE}$$

$$\frac{a}{3.2} = \frac{2}{5} \qquad \frac{b}{6} = \frac{2}{5}$$

$$3.2\left(\frac{a}{3.2}\right) = 3.2\left(\frac{2}{5}\right) \qquad 6\left(\frac{b}{6}\right) = 6\left(\frac{2}{5}\right)$$

$$a = 1.28 \text{ cm} \qquad b = 2.4 \text{ cm}$$

b. The following is one way to solve this question.

$$\frac{ST}{YZ} = \frac{RS}{XY} \qquad \frac{XZ}{RT} = \frac{XY}{RS}$$

$$\frac{r}{1.5} = \frac{4}{2.5} \qquad \frac{y}{5} = \frac{2.5}{4}$$

$$1.5\left(\frac{r}{1.5}\right) = 1.5\left(\frac{4}{2.5}\right) \qquad 5\left(\frac{y}{5}\right) = 5\left(\frac{2.5}{4}\right)$$

$$r = 2.4 \text{ cm} \qquad y = 3.125 \text{ cm}$$

8. Yes. Congruent triangles are always similar because they are identical in shape and size.

9. No. Similar triangles are not always congruent because they are identical in shape (congruent corresponding angles) but not always identical in size (proportional corresponding sides).

10. Let h represent Joanna's height.

$$\therefore \frac{h}{8} = \frac{2.2}{10}$$

$$8\left(\frac{h}{8}\right) = 8\left(\frac{2.2}{10}\right)$$

$$h = 1.76$$

Joanna is 1.76 m tall.

11.

$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{x+6}{6} = \frac{26}{8}$$

$$24\left(\frac{x+6}{6}\right) = 24\left(\frac{26}{8}\right)$$

$$4x + 24 = 78$$

$$4x + 24 - 24 = 78 - 24$$

$$4x = 54$$

$$\frac{4x}{4} = \frac{54}{4}$$

$$x = 13.5 \text{ cm}$$

12. Yes. The angles are always 90°, and the corresponding sides are always proportional.

13. a. All three triangles share a 30° angle, and each triangle has a 90° angle; therefore, the remaining angle in each of these triangles is 60°. Thus, the three triangles are similar because their corresponding angles are congruent.

b. ΔAEK, ΔAFL, and ΔAGM

c. $BH = 1$ cm and $AB = 2$ cm

$$\frac{BH}{AB} = \frac{1}{2}$$

d. $CI = 1.5$ cm and $AC = 3$ cm

$$\frac{CI}{AC} = \frac{1.5}{3}$$
$$= \frac{1}{2}$$

e. The ratios are all $\frac{1}{2}$ because they are corresponding sides of similar triangles and, therefore, must be proportional (have the same ratio).

f.
$$\frac{XY}{AX} = \frac{1}{2}$$
$$\frac{9.5}{AX} = \frac{1}{2}$$
$$9.5 \times 2 = 1 \times AX$$
$$19 = AX$$
$$AX = 19 \text{ cm}$$

g. The ratio of the length of the side opposite the 30° angle to the length of the side opposite the 90° angle (or hypotenuse) is $\frac{1}{2}$.

14. a.
$$\frac{EB}{AE} = \frac{1}{2}$$
$$\frac{EB}{600} = \frac{1}{2}$$
$$\cancel{600}\left(\frac{EB}{\cancel{600}}\right) = \overset{300}{\cancel{600}}\left(\frac{1}{\cancel{2}_{1}}\right)$$
$$EB = 300$$

Fredina and Maurice are 300 m from the riverbank.

b. The river is 600 m wide.

c. AC is the distance that the canoe floats downstream while crossing the river. To find AC, you need to find AB first. Use the Pythagorean Theorem to calculate AB.

$$(AB)^2 + (EB)^2 = (AE)^2$$
$$(AB)^2 + 300^2 = 600^2$$
$$(AB)^2 + 90\,000 = 360\,000$$
$$(AB)^2 + 90\,000 - 90\,000 = 360\,000 - 90\,000$$
$$(AB)^2 = 270\,000$$
$$AB = \sqrt{270\,000}$$
$$\doteq 519.6 \text{ m}$$

Now find AC.

$$\frac{AC}{AB} = \frac{DC}{EB}$$
$$\frac{AC}{519.6} \doteq \frac{600}{300}$$
$$\frac{AC}{519.6} \doteq 2$$
$$519.6\left(\frac{AC}{519.6}\right) \doteq 519.6(2)$$
$$AC \doteq 1039.2$$

Fredina and Maurice should begin crossing about 1039 m upstream from the spot on the opposite riverbank where they wish to land.

Looking Back

15. a.

$$\frac{x}{1} = \frac{x+3}{2}$$
$$2\left(\frac{x}{1}\right) = 2\left(\frac{x+3}{2}\right)$$
$$2x = x+3$$
$$2x - x = x+3-x$$
$$x = 3$$

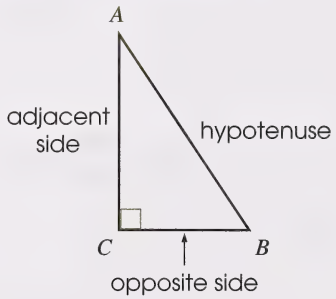
b. Let h represent the height of the tree.

$$\therefore \frac{h}{3+25} = \frac{2}{3+3}$$
$$\frac{h}{28} = \frac{2}{6}$$
$$\frac{h}{28} = \frac{1}{3}$$
$$28\left(\frac{h}{28}\right) = 28\left(\frac{1}{3}\right)$$
$$h \doteq 9.3$$

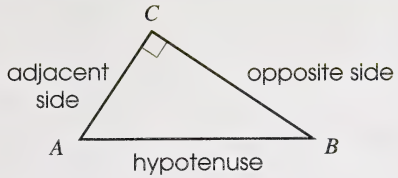
The tree is approximately 9.3 m tall.

Section 2: Activity 4

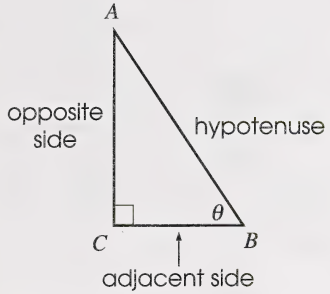
1. a.



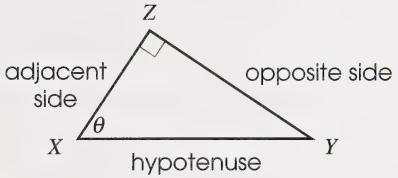
b.



2. a.



b.



3. a. $\frac{BC}{AC} = \frac{3}{4}$ $= 0.75$ $\quad \frac{DE}{AE} = \frac{6}{8}$ $= 0.75$ $\quad \frac{FG}{AG} = \frac{9}{12}$ $= 0.75$

b. Yes, the ratio is the same for each triangle.

4. The measure of $\angle A$ is approximately 37°.

5.

This value is approximately 0.7536.

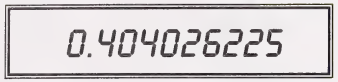
Note: If the value you obtained was about 0.8408, then your calculator was in radian mode. You need to switch to degree mode.

6. This value is close to the value of the tangent ratio that was obtained previously.

7. a.

$\therefore$ tan 81° ≐ 6.3138

b. tan 2 2 =

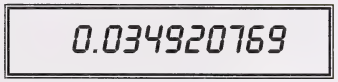


$\therefore \tan 22° \doteq 0.4040$

c. tan 4 5 =

$\therefore \tan 45° = 1$

d. tan 2 =



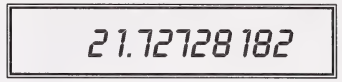
$\therefore \tan 2° \doteq 0.0349$

8. a. $\tan\theta = \frac{12}{5}$ or 2.4 $\qquad \tan\alpha = \frac{5}{12}$ or 0.4167

b. $\tan\theta = \frac{36}{27}$ $\qquad \tan\alpha = \frac{27}{36}$

$= \frac{4}{3}$ or 1.3333 $\qquad = \frac{3}{4}$ or 0.75

9. a. 2nd tan • 3 9 8 5 =

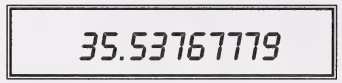


$\therefore \angle\theta \doteq 22°$

b. 2nd tan 9 • 8 4 7 6 =

$\therefore \angle\theta \doteq 84°$

c. 2nd tan (5 ÷ 7) =



$\therefore \angle\theta \doteq 36°$

d. 2nd tan (4 ÷ 3) =

$\therefore \angle\theta \doteq 53°$

10. $\tan D = \dfrac{\text{opp}}{\text{adj}}$

$= \dfrac{EF}{DE}$

$= \dfrac{4}{10}$

$= \dfrac{2}{5}$

21.80140949

$\therefore \angle D \doteq 22°$

$\tan F = \dfrac{\text{opp}}{\text{adj}}$

$= \dfrac{DE}{EF}$

$= \dfrac{10}{4}$

$= \dfrac{5}{2}$

68.19859051

$\therefore \angle F \doteq 68°$

11. a. $\tan 65° = \dfrac{\text{opp}}{\text{adj}}$

$\tan 65° = \dfrac{XY}{3}$

$XY = 3 \tan 65°$

6.433520762

The length of XY is about 6.4 m.

b. Method 1

$\angle X = 90° - 65°$

$= 25°$

Method 2

$\tan X = \dfrac{\text{opp}}{\text{adj}}$

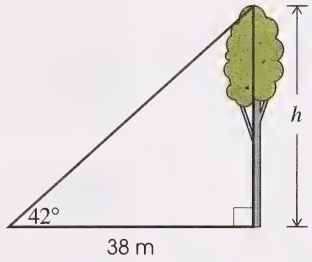
$\tan X \doteq \dfrac{3}{6.4}$

25.11483489

$\therefore \angle X \doteq 25°$

Note: The answers from the two methods differ slightly because the rounded value of the adjacent side was used in the calculation in Method 2.

12. Draw a diagram to illustrate the problem.



$$\tan 42° = \frac{h}{38}$$

$$38(\tan 42°) = \cancel{38}\left(\frac{h}{\cancel{38}}\right)$$

$$38 \tan 42° = h$$

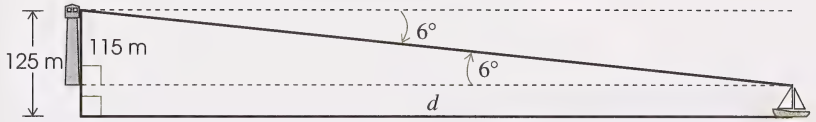
$$h = 38 \tan 42°$$

3 8 × tan 4 2 =

34.2 1535368

The height of the tree is about 34 m.

13. Draw a diagram.



$$\tan 6° = \frac{115}{d}$$

$$d(\tan 6°) = \cancel{d}\left(\frac{115}{\cancel{d}}\right)$$

$$d \tan 6° = 115$$

$$\frac{d \tan 6°}{\tan 6°} = \frac{115}{\tan 6°}$$

$$d = \frac{115}{\tan 6°}$$

1 1 5 ÷ tan 6 =

1094. 15 19 12

The sailboat is about 1094 m (or 1.09 km) from shore.

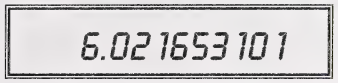
14. $\tan 16° = \dfrac{\text{opp}}{\text{adj}}$

$\tan 16° = \dfrac{x}{21}$

$21(\tan 16°) = 21\left(\dfrac{x}{21}\right)$

$21 \tan 16° = x$

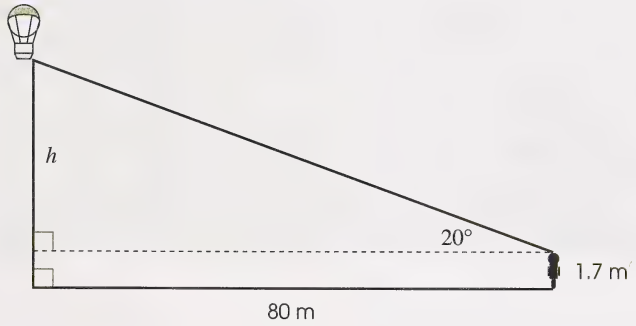
$x = 21 \tan 16°$



The height of the tree is approximately $6.0 + 1.6 = 7.6$ m.

15. A reasonable answer may range from a few metres to 100 m or more.

16. Draw a diagram.



$\tan 20° = \dfrac{h}{80}$

$80(\tan 20°) = 80\left(\dfrac{h}{80}\right)$

$80 \tan 20° = h$

$h = 80 \tan 20°$

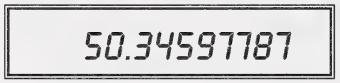
The height of the inflated hot-air balloon is approximately $29.1 + 1.7 = 30.8$ m.

17. $\tan 40° = \dfrac{h}{60}$

$60(\tan 40°) = 60\left(\dfrac{h}{60}\right)$

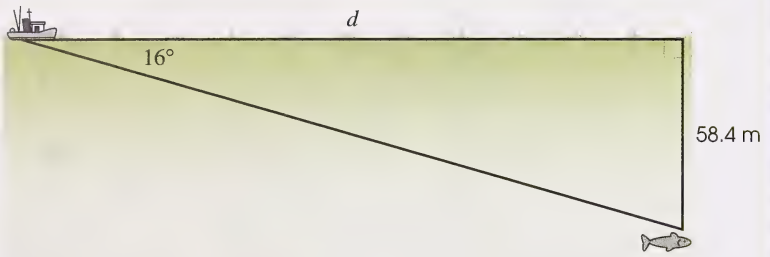
$60 \tan 40° = h$

$h = 60 \tan 40°$



The height of the building is about 50.3 m.

18. Draw a diagram.



$$\tan 16° = \frac{58.4}{d}$$

$$d(\tan 16°) = \cancel{d}\left(\frac{58.4}{\cancel{d}}\right)$$

$$d \tan 16° = 58.4$$

$$\frac{d \tan 16°}{\tan 16°} = \frac{58.4}{\tan 16°}$$

$$d = \frac{58.4}{\tan 16°}$$

5 8 • 4 ÷ tan 1 6 =

203.6650035

The captain must move the schooner about 204 m in order to be directly over the school of fish.

19. a.

Angle	Tangent	Angle	Tangent
5°	0.0875	50°	1.1918
10°	0.1763	55°	1.4281
15°	0.2679	60°	1.7321
20°	0.3640	65°	2.1445
25°	0.4663	70°	2.7475
30°	0.5774	75°	3.7321
35°	0.7002	80°	5.6713
40°	0.8391	85°	11.4301
45°	1.000	90°	Error

b. The value of the tangent ratio ranges from almost 0 when the angle is 5° to over 11 when the angle is 85°.

c. From the pattern in question 19.a., it appears the tangent value for 0° will be 0.

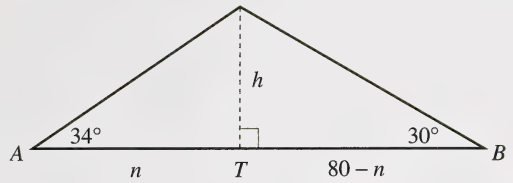
tan 0 =

0.

d. As the angle approaches 90°, the length of the side you are dividing by approaches 0. At 90°, the length of the side you are dividing by is 0; and division by zero is undefined.

Now Try This

20. Draw a diagram.



Let h be the height of the tree, and let n be the distance from A to the tree. Therefore, $80 - n$ is the distance from B to the tree.

$$\tan A = \frac{h}{AT}$$

$$\tan 34° = \frac{h}{n}$$

$$n(\tan 34°) = n\left(\frac{h}{n}\right)$$

$$n \tan 34° = h$$

$$h = n \tan 34°$$

$$\doteq 0.6745n$$

$$\tan B = \frac{h}{BT}$$

$$\tan 30° = \frac{h}{80 - n}$$

$$(80 - n)(\tan 30°) = (80 - n)\left(\frac{h}{80 - n}\right)$$

$$(80 - n)\tan 30° = h$$

$$h = (80 - n)\tan 30°$$

$$\doteq (80 - n)(0.5774)$$

$$\doteq 46.1880 - 0.5774n$$

Since $h = h$, then

$$0.6745n \doteq 46.1880 - 0.5774n$$

$$0.6745n + 0.5774n \doteq 46.1880 - 0.5774n + 0.5774n$$

$$1.2519n \doteq 46.1880$$

$$\frac{1.2519n}{1.2519} \doteq \frac{46.1880}{1.2519}$$

$$n \doteq 37$$

Substitute $n \doteq 37$ into one of the equations of h.

$$0.6745n \doteq 0.6745(37)$$

$$\doteq 25$$

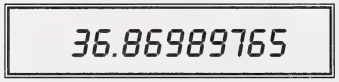
The height of the tree is about 25 m.

21. a. $\frac{BC}{AB} = \frac{3}{5}$ or 0.6 $\qquad \frac{DE}{AD} = \frac{6}{10}$ or 0.6

$$\frac{FG}{AF} = \frac{9}{12} \text{ or } 0.6$$

b. Yes, the ratios are the same for each triangle.

22.



This is 37° rounded to the nearest degree. This value is $\angle A$.

23. a. $\frac{AC}{AB} = \frac{4}{5}$ or 0.8 $\quad \frac{AE}{AD} = \frac{8}{10}$ or 0.8

$\frac{AG}{AF} = \frac{12}{15}$ or 0.8

b. Yes, the ratios are the same for each triangle.

24. [2nd] [cos] [(] [4] [÷] [5] [)] [=]

36.86989765

The resulting angle is 37°, rounded to the nearest degree.

25. The measure of $\angle A$ is the same as the values obtained using the tangent and sine ratios.

26. a. [sin] [3] [0] [=]

0.5

[cos] [3] [0] [=]

0.866025403

$\therefore$ $\sin 30° = 0.5$ and $\cos 30° \doteq 0.8660$

b. [sin] [8] [7] [=]

0.998629534

[cos] [8] [7] [=]

0.052335956

$\therefore$ $\sin 87° \doteq 0.9986$ and $\cos 87° \doteq 0.0523$

c. [sin] [9] [=]

0.156434465

[cos] [9] [=]

0.98768834

$\therefore$ $\sin 9° \doteq 0.1564$ and $\cos 9° \doteq 0.9877$

d. sin 4 5 =

cos 4 5 =

$\therefore \sin 45° \doteq 0.7071$ and $\cos 45° \doteq 0.7071$

e. sin 6 0 =

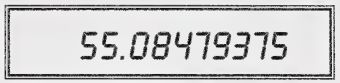
cos 6 0 =

$\therefore \sin 60° \doteq 0.8660$ and $\cos 60° \doteq 0.5$

27. a. $\sin 0° = 0$, $\sin 90° = 1$, $\cos 0° = 1$, and $\cos 90° = 0$.

b. The sine and cosine ratios are the same when the angle is 45°.

28. a. 2nd sin • 8 2 =



$\therefore \angle C \doteq 55°$

b. 2nd cos • 0 8 1 4 =

$\therefore \angle E \doteq 85°$

c. 2nd sin (5 ÷ 8) =

$\therefore \angle A \doteq 39°$

d. 2nd cos (1 6 ÷ 2 3) =

$\therefore \angle B \doteq 46°$

29. a. $\sin A = \frac{\text{opp}}{\text{hyp}}$ $\qquad$ $\cos A = \frac{\text{adj}}{\text{hyp}}$

$= \frac{30}{34}$ $\qquad$ $= \frac{16}{34}$

$\doteq 0.8824$ $\qquad$ $\doteq 0.4706$

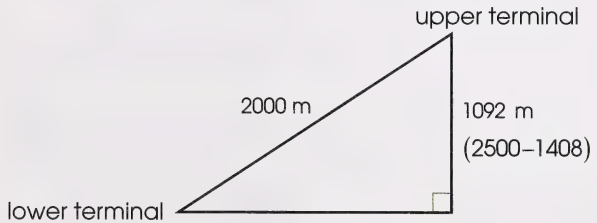
$\sin B = \frac{\text{opp}}{\text{hyp}}$ $\qquad$ $\cos B = \frac{\text{adj}}{\text{hyp}}$

$= \frac{16}{34}$ $\qquad$ $= \frac{30}{34}$

$\doteq 0.4706$ $\qquad$ $\doteq 0.8824$

b. The ratios for $\sin A$ and $\cos B$ are the same. The ratios for $\sin B$ and $\cos A$ are also the same.

30. Draw a diagram.



$\sin \theta = \frac{\text{opp}}{\text{hyp}}$

$= \frac{1092}{2000}$

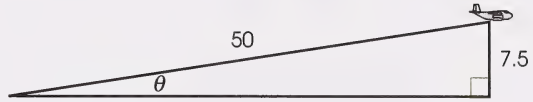
$= \frac{273}{500}$

2nd sin (2 7 3 ÷ 5 0 0) =

33.09302665

The angle of inclination is about 33°.

31. Draw a diagram.



$\sin \theta = \frac{\text{opp}}{\text{hyp}}$

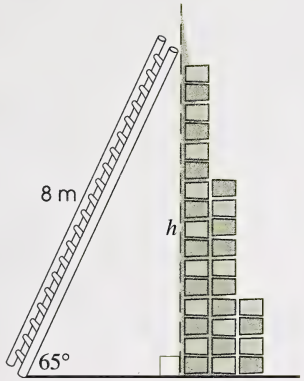
$= \frac{7.5}{50}$

2nd sin (7 • 5 ÷ 5 0) =

8.626926559

The airplane is climbing at an angle of about 9°.

32. Draw a diagram.



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 65° = \frac{h}{8}$$

$$8(\sin 65°) = \cancel{8}\left(\frac{h}{\cancel{8}}\right)$$

$$8 \sin 65° = h$$

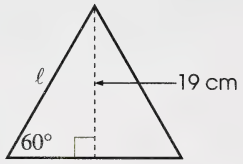
$$h = 8 \sin 65°$$

8 × sin 6 5 =

7.250462296

The ladder will reach about 7.25 m up the wall.

33. Draw a diagram.



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 60° = \frac{19}{\ell}$$

$$\ell(\sin 60°) = \cancel{\ell}\left(\frac{19}{\cancel{\ell}}\right)$$

$$\ell \sin 60° = 19$$

$$\frac{\ell \sin 60°}{\sin 60°} = \frac{19}{\sin 60°}$$

$$\ell = \frac{19}{\sin 60°}$$

1 9 ÷ sin 6 0 =

21.93931023

Each side of the triangle is approximately 22 cm long.

34. First find a.

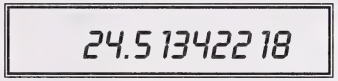
$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 40° = \frac{a}{32}$$

$$32(\cos 40°) = 32\left(\frac{a}{32}\right)$$

$$32\cos 40° = a$$

$$a = 32\cos 40°$$



$\therefore\ a \doteq 25$ cm

You must find the measure of x before determining the value of b.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 40° = \frac{x}{32}$$

$$32(\sin 40°) = 32\left(\frac{x}{32}\right)$$

$$32\sin 40° = x$$

$$x = 32\sin 40°$$

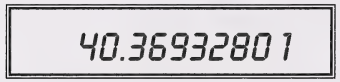
$\therefore\ x \doteq 20.5692$

Now find b.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 27° \doteq \frac{20.5692}{b}$$

$$b \doteq \frac{20.5692}{\tan 27°}$$

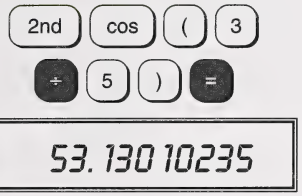


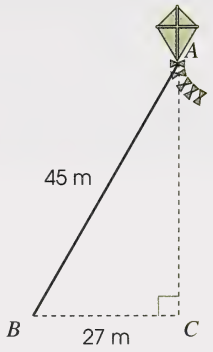
Therefore, the length of side a is about 25 cm, and the length of side b is about 40 cm.

35. Draw a diagram.

a. $\cos B = \frac{27}{45}$

$= \frac{3}{5}$

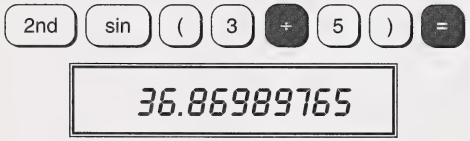




The angle the kite's string formed with the ground was about 53°.

b. $\sin A = \frac{27}{45}$

$= \frac{3}{5}$



The angle the kite's string formed with a vertical line from the kite to the ground is about 37°.

Note: You may also use $90° - 53° \doteq 37°$ to solve this question.

c. You could have used one of several methods to determine the height of the kite. One method is using the Pythagorean Theorem.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (27)^2 + b^2 &= (45)^2 \\ 729 + b^2 &= 2025 \\ 729 + b^2 - 729 &= 2025 - 729 \\ b^2 &= 1296 \\ b &= \sqrt{1296} \\ &= 36 \end{aligned}$$

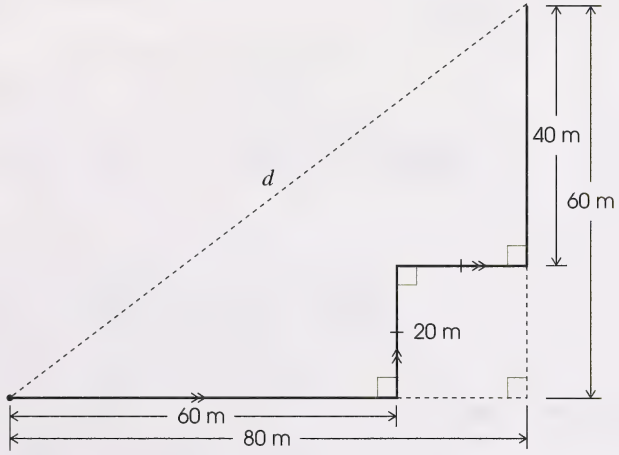
The kite was flying 36 m above the ground.

d. Answers will vary depending on the method you used in question 35.c. Methods to determine the height of the kite include the following:

- the Pythagorean Theorem (as used in question 35.c.)
- the tangent of angle B with the height as the opposite side and 27 m as the adjacent side
- the sine of angle B with the height as the opposite side and the hypotenuse as the other side
- the tangent of angle A with 27 m as the opposite side and the height as the adjacent side
- the cosine of angle A with the height as the adjacent side and the hypotenuse as the other side

Now Try This

36. Draw a diagram to illustrate the problem.



Use the Pythagorean Theorem to solve this problem.

$$d^2 = (80)^2 + (60)^2$$
$$d^2 = 6400 + 3600$$
$$d^2 = 10\,000$$
$$d = \sqrt{10\,000}$$
$$= 100$$

Shawna is 100 m from her starting point.

Looking Back

37. SOH CAH TOA represents the trigonometric ratios. The sine ratio is opposite over hypotenuse; the cosine ratio is adjacent over hypotenuse; and the tangent ratio is opposite over adjacent.

38. $\sin\theta = \frac{g}{h}$ $\cos\theta = \frac{i}{h}$ $\tan\theta = \frac{g}{i}$

39. 2nd sin • 7 7 =

50.35388885

Note: Make sure your calculator is in degree mode.

Section 2: Activity 5

1. a. $AB = 2.5$ cm $A'B' = 5.0$ cm
$AC = 2.5$ cm $A'C' = 5.0$ cm
$BC = 1.8$ cm $B'C' = 3.6$ cm

The lengths of the sides of $\Delta A'B'C'$ are twice those of ΔABC.

b. $\angle A = 42°$ $\angle A' = 42°$
$\angle B = 68°$ $\angle B' = 68°$
$\angle C = 70°$ $\angle C' = 70°$

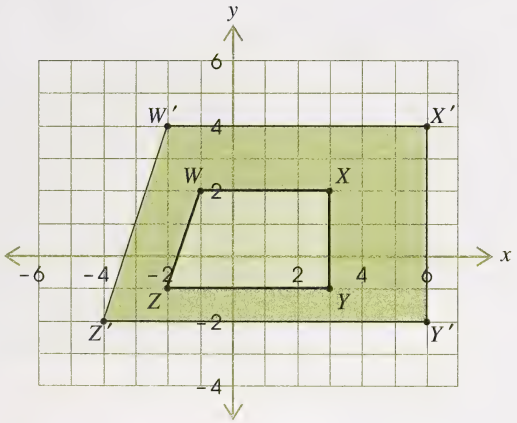
The corresponding angles of ΔABC and $\Delta A'B'C'$ have the same measure.

c. The corresponding sides are parallel.

2. The scale factor is 2.

3. $(x, y) \rightarrow (2x, 2y)$

4. The vertices of the image of $WXYZ$ are $W'(-2, 4)$, $X'(6, 4)$, $Y'(6, -2)$, and $Z'(-4, -2)$.



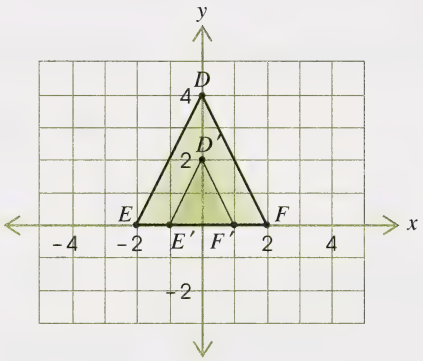
5. a. Area of $\Delta ABC = \frac{1}{2}(4)(3)$
$= 6$ square units

Area of $\Delta A'B'C' = \frac{1}{2}(8)(6)$
$= 24$ square units

b. The area of $\Delta A'B'C'$ is equal to the scale factor squared times the area of ΔABC. That is,

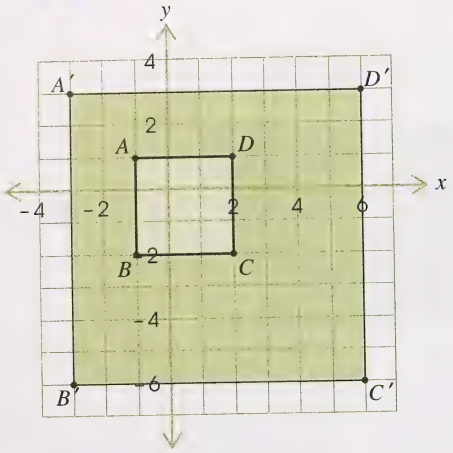
$24 = 2^2 \times 6$

6. a.



b. $(x, y) = \left(\frac{1}{2}x, \frac{1}{2}y\right)$

7.



8. **a.** The coordinates of the vertices and the lengths of the corresponding sides change.

b. The measure of the corresponding angles, the orientation of the figure, and the slope of the sides remain the same.

9.

Original	Image	Mapping Rule
$(3, -1)$	$(6, -2)$	$(x, y) \rightarrow (2x, 2y)$
$(4, 2)$	$(2, 1)$	$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$
$(-4, 6)$	$(-2, 3)$	$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$
$(4, 6)$	$(6, 9)$	$(x, y) \rightarrow (1.5x, 1.5y)$
$(3, 2)$	$(9, 6)$	$(x, y) \rightarrow (3x, 3y)$

10.

Area of Original	Area of Image	Scale Factor
24 cm^2	6 cm^2	$\frac{1}{2}$
16 cm^2	36 cm^2	1.5
36 cm^2	4 cm^2	$\frac{1}{3}$
8 cm^2	50 cm^2	2.5

11. **a.** The scale factor is 4. **b.** The scale factor is $\frac{1}{2}$.

Now Try This

12. Answers will vary. Some examples of real-world dilatations are as follows:

- a model airplane, which is a reduction
- the large Easter egg in Vegreville, Alberta, which is an enlargement
- a statue of a person, which may be an enlargement or a reduction of the actual person

Looking Back

13. If $0 < k < 1$, the new shape would be reduced. When a number is multiplied by a fraction between 0 and 1, the resulting product is smaller than the original number. For example, $5 \times \frac{2}{3} = \frac{10}{3}$, which is smaller than 5.

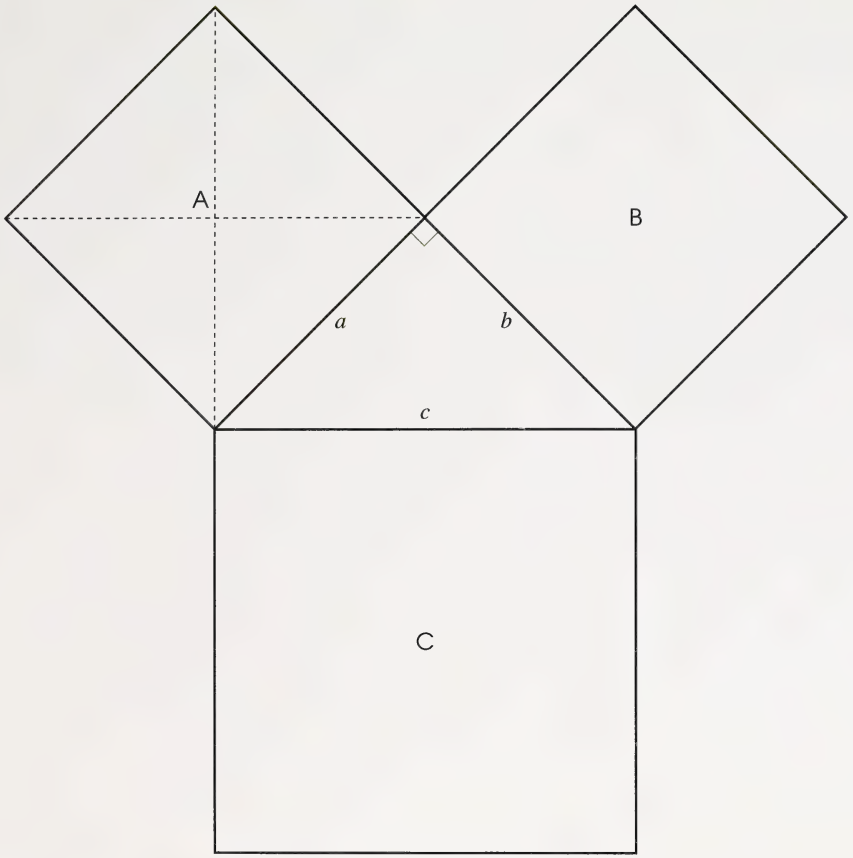
Credits

Some clip art drawings are commercially owned.

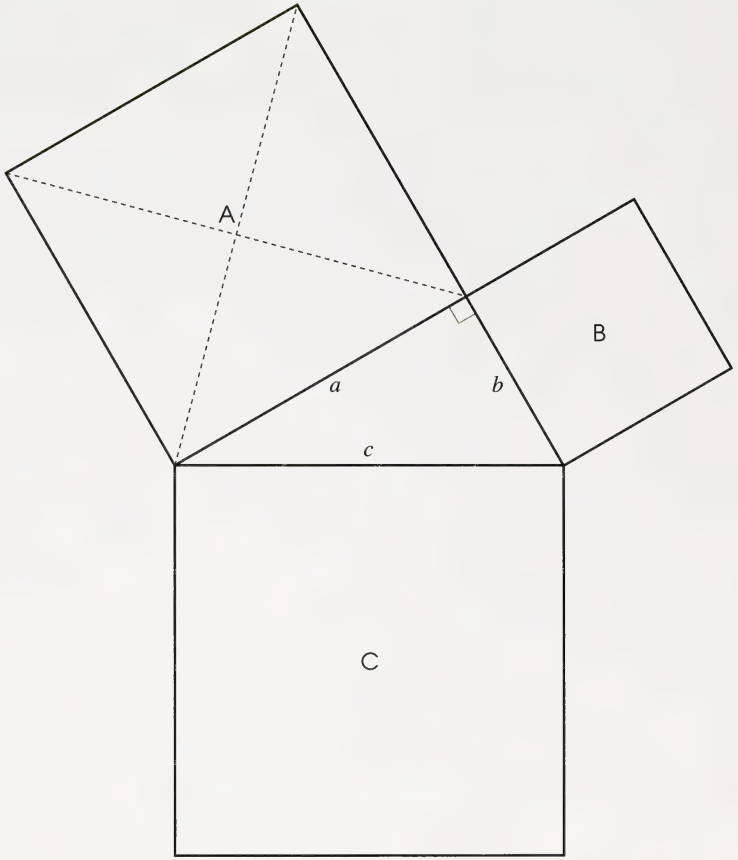
Welcome Page
PhotoDisc, Inc.

Page
1 PhotoDisc, Inc.
3 Corel Corporation
4 **left:** Corel Corporation
right: PhotoDisc, Inc.
5 PhotoDisc, Inc.
7 **upper left:** PhotoDisc, Inc.
lower left: PhotoDisc, Inc.
right: PhotoDisc, Inc.
8 PhotoDisc, Inc.
11 **upper right:** PhotoDisc, Inc.
12 PhotoDisc, Inc.
13 **right:** Corel Corporation
14 **right:** Corel Corporation
15 **left:** PhotoDisc, Inc.
16 **upper left:** PhotoDisc, Inc.
upper right: PhotoDisc, Inc.
19 **left:** PhotoDisc, Inc.
20 PhotoDisc, Inc.
21 EyeWire, Inc.
22 Image Club/StudioGear/EyeWire, Inc.
24 PhotoDisc, Inc.
25 PhotoDisc, Inc.
26 PhotoDisc, Inc.
27 **left:** EyeWire, Inc.
29 PhotoDisc, Inc.
30 Corbis
31 **left:** Corel Corporation
right: PhotoDisc, Inc.
38 Corel Corporation
40 PhotoDisc, Inc.
45 **left:** PhotoDisc, Inc.
right: PhotoDisc, Inc.
48 **upper right:** Digital Vision Ltd.
51 **upper right:** PhotoDisc, Inc.
lower right: PhotoDisc, Inc.
53 PhotoDisc, Inc.
54 PhotoDisc, Inc.
55 PhotoDisc, Inc.
59 **left:** PhotoDisc, Inc.
right: PhotoDisc, Inc.
60 **left:** EyeWire, Inc.
right: PhotoDisc, Inc.
61 **lower left:** PhotoDisc, Inc.
64 **left:** PhotoDisc, Inc.
right: PhotoDisc, Inc.
65 **left:** Corel Corporation
right: EyeWire, Inc.
68 PhotoDisc, Inc.
77 PhotoDisc, Inc.
78 **left:** PhotoDisc, Inc.
79 PhotoDisc, Inc.

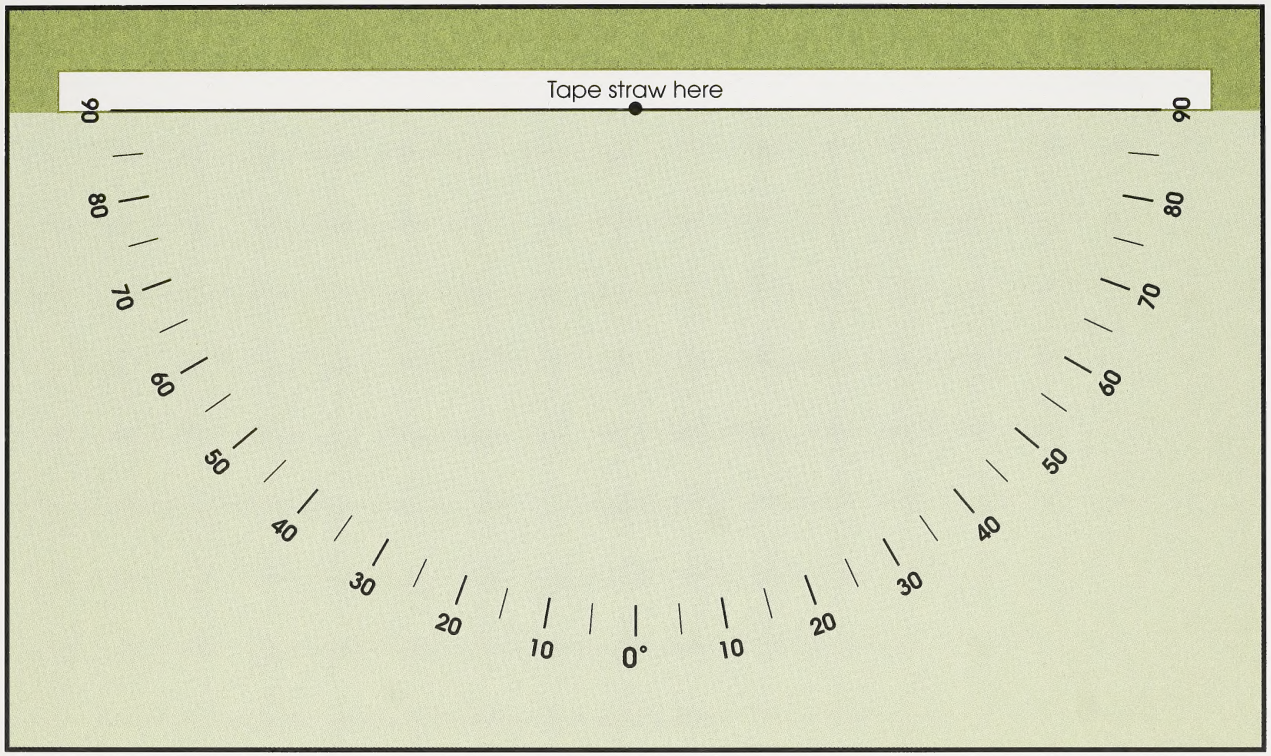
Pythagorean Puzzle 1

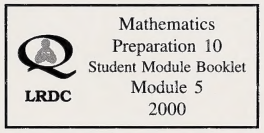


Pythagorean Puzzle 2



Clinometer


Tape straw here
90
80
70
60
50
40
30
20
10
0°
10
20
30
40
50
60
70
80
90



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